

Maple 2018.2 Integration Test Results
on the problems in "1 Algebraic functions/1.2 Trinomial products/1.2.3 General"

Test results for the 179 problems in "1.2.3.2 (d x)^m (a+b x^n+c x^(2 n))^p.txt"

Problem 36: Unable to integrate problem.

$$\int \frac{(dx)^m}{(b^2 x^6 + 2 a b x^3 + a^2)^{3/2}} dx$$

Optimal(type 5, 60 leaves, 2 steps):

$$\frac{(dx)^{1+m} (bx^3 + a) \operatorname{hypergeom}\left(\left[3, \frac{1}{3} + \frac{m}{3}\right], \left[\frac{4}{3} + \frac{m}{3}\right], -\frac{bx^3}{a}\right)}{a^3 d (1+m) \sqrt{(bx^3 + a)^2}}$$

Result(type 8, 28 leaves):

$$\int \frac{(dx)^m}{(b^2 x^6 + 2 a b x^3 + a^2)^{3/2}} dx$$

Problem 37: Unable to integrate problem.

$$\int (dx)^m (b^2 x^6 + 2 a b x^3 + a^2)^p dx$$

Optimal(type 5, 77 leaves, 2 steps):

$$\frac{(dx)^{1+m} (b^2 x^6 + 2 a b x^3 + a^2)^p \operatorname{hypergeom}\left(\left[-2p, \frac{1}{3} + \frac{m}{3}\right], \left[\frac{4}{3} + \frac{m}{3}\right], -\frac{bx^3}{a}\right)}{d (1+m) \left(1 + \frac{bx^3}{a}\right)^{2p}}$$

Result(type 8, 28 leaves):

$$\int (dx)^m (b^2 x^6 + 2 a b x^3 + a^2)^p dx$$

Problem 39: Unable to integrate problem.

$$\int x (b^2 x^6 + 2 a b x^3 + a^2)^p dx$$

Optimal(type 5, 54 leaves, 2 steps):

$$\frac{x^2 (bx^3 + a) (b^2 x^6 + 2 a b x^3 + a^2)^p \operatorname{hypergeom}\left(\left[1, \frac{5}{3} + 2p\right], \left[\frac{5}{3}\right], -\frac{bx^3}{a}\right)}{2 a}$$

Result(type 8, 24 leaves):

$$\int x (b^2 x^6 + 2 a b x^3 + a^2)^p dx$$

Problem 40: Unable to integrate problem.

$$\int \frac{(b^2 x^6 + 2 a b x^3 + a^2)^p}{x} dx$$

Optimal(type 5, 63 leaves, 3 steps):

$$\frac{(b x^3 + a) (b^2 x^6 + 2 a b x^3 + a^2)^p \operatorname{hypergeom}\left(\left[1, 1 + 2 p\right], \left[2 + 2 p\right], 1 + \frac{b x^3}{a}\right)}{3 a (1 + 2 p)}$$

Result(type 8, 26 leaves):

$$\int \frac{(b^2 x^6 + 2 a b x^3 + a^2)^p}{x} dx$$

Problem 42: Result is not expressed in closed-form.

$$\int \frac{x}{c x^6 + b x^3 + a} dx$$

Optimal(type 3, 421 leaves, 13 steps):

$$\begin{aligned} & - \frac{2^{1/3} c^{1/3} \ln\left(2^{1/3} c^{1/3} x + (b - \sqrt{-4ac + b^2})^{1/3}\right)}{3 (b - \sqrt{-4ac + b^2})^{1/3} \sqrt{-4ac + b^2}} + \frac{c^{1/3} \ln\left(2^{2/3} c^{2/3} x^2 - 2^{1/3} c^{1/3} x (b - \sqrt{-4ac + b^2})^{1/3} + (b - \sqrt{-4ac + b^2})^{2/3}\right) 2^{1/3}}{6 (b - \sqrt{-4ac + b^2})^{1/3} \sqrt{-4ac + b^2}} \\ & - \frac{2^{1/3} c^{1/3} \arctan\left(\frac{\left(1 - \frac{2^{2/3} c^{1/3} x}{(b - \sqrt{-4ac + b^2})^{1/3}}\right) \sqrt{3}}{3}\right) \sqrt{3}}{3 (b - \sqrt{-4ac + b^2})^{1/3} \sqrt{-4ac + b^2}} + \frac{2^{1/3} c^{1/3} \ln\left(2^{1/3} c^{1/3} x + (b + \sqrt{-4ac + b^2})^{1/3}\right)}{3 \sqrt{-4ac + b^2} (b + \sqrt{-4ac + b^2})^{1/3}} \\ & - \frac{c^{1/3} \ln\left(2^{2/3} c^{2/3} x^2 - 2^{1/3} c^{1/3} x (b + \sqrt{-4ac + b^2})^{1/3} + (b + \sqrt{-4ac + b^2})^{2/3}\right) 2^{1/3}}{6 \sqrt{-4ac + b^2} (b + \sqrt{-4ac + b^2})^{1/3}} \\ & + \frac{2^{1/3} c^{1/3} \arctan\left(\frac{\left(1 - \frac{2^{2/3} c^{1/3} x}{(b + \sqrt{-4ac + b^2})^{1/3}}\right) \sqrt{3}}{3}\right) \sqrt{3}}{3 \sqrt{-4ac + b^2} (b + \sqrt{-4ac + b^2})^{1/3}} \end{aligned}$$

Result(type 7, 40 leaves):

$$\left(\frac{\sum_{R=\text{RootOf}(c Z^6+b Z^3+a)} \frac{R \ln(x-R)}{2 R^5 c + R^2 b}}{3} \right)$$

Problem 43: Result is not expressed in closed-form.

$$\int \frac{1}{c x^6 + b x^3 + a} dx$$

Optimal(type 3, 421 leaves, 13 steps):

$$\begin{aligned} & \frac{2^{2/3} c^{2/3} \ln\left(2^{1/3} c^{1/3} x + \left(b - \sqrt{-4ac + b^2}\right)^{1/3}\right)}{3 \left(b - \sqrt{-4ac + b^2}\right)^{2/3} \sqrt{-4ac + b^2}} - \frac{c^{2/3} \ln\left(2^{2/3} c^{2/3} x^2 - 2^{1/3} c^{1/3} x \left(b - \sqrt{-4ac + b^2}\right)^{1/3} + \left(b - \sqrt{-4ac + b^2}\right)^{2/3}\right) 2^{2/3}}{6 \left(b - \sqrt{-4ac + b^2}\right)^{2/3} \sqrt{-4ac + b^2}} \\ & - \frac{2^{2/3} c^{2/3} \arctan\left(\frac{\left(1 - \frac{2^{2^{1/3}} c^{1/3} x}{\left(b - \sqrt{-4ac + b^2}\right)^{1/3}}\right) \sqrt{3}}{3}\right) \sqrt{3}}{3 \left(b - \sqrt{-4ac + b^2}\right)^{2/3} \sqrt{-4ac + b^2}} - \frac{2^{2/3} c^{2/3} \ln\left(2^{1/3} c^{1/3} x + \left(b + \sqrt{-4ac + b^2}\right)^{1/3}\right)}{3 \sqrt{-4ac + b^2} \left(b + \sqrt{-4ac + b^2}\right)^{2/3}} \\ & + \frac{c^{2/3} \ln\left(2^{2/3} c^{2/3} x^2 - 2^{1/3} c^{1/3} x \left(b + \sqrt{-4ac + b^2}\right)^{1/3} + \left(b + \sqrt{-4ac + b^2}\right)^{2/3}\right) 2^{2/3}}{6 \sqrt{-4ac + b^2} \left(b + \sqrt{-4ac + b^2}\right)^{2/3}} \\ & + \frac{2^{2/3} c^{2/3} \arctan\left(\frac{\left(1 - \frac{2^{2^{1/3}} c^{1/3} x}{\left(b + \sqrt{-4ac + b^2}\right)^{1/3}}\right) \sqrt{3}}{3}\right) \sqrt{3}}{3 \sqrt{-4ac + b^2} \left(b + \sqrt{-4ac + b^2}\right)^{2/3}} \end{aligned}$$

Result(type 7, 39 leaves):

$$\left(\frac{\sum_{R=\text{RootOf}(c Z^6+b Z^3+a)} \frac{\ln(x-R)}{2 R^5 c + R^2 b}}{3} \right)$$

Problem 49: Unable to integrate problem.

$$\int x^5 \sqrt{c x^6 + b x^3 + a} dx$$

Optimal(type 3, 90 leaves, 5 steps):

$$\frac{(c x^6 + b x^3 + a)^{3/2}}{9c} + \frac{b(-4ac + b^2) \operatorname{arctanh}\left(\frac{2cx^3 + b}{2\sqrt{c}\sqrt{cx^6 + bx^3 + a}}\right)}{48c^{5/2}} - \frac{b(2cx^3 + b)\sqrt{cx^6 + bx^3 + a}}{24c^2}$$

Result(type 8, 80 leaves):

$$\frac{(8c^2x^6 + 2bcx^3 + 8ac - 3b^2)\sqrt{cx^6 + bx^3 + a}}{72c^2} + \int -\frac{b(4ac - b^2)x^2}{16c^2\sqrt{cx^6 + bx^3 + a}} dx$$

Problem 50: Unable to integrate problem.

$$\int \frac{\sqrt{cx^6 + bx^3 + a}}{x} dx$$

Optimal(type 3, 85 leaves, 7 steps):

$$-\frac{\operatorname{arctanh}\left(\frac{bx^3 + 2a}{2\sqrt{a}\sqrt{cx^6 + bx^3 + a}}\right)\sqrt{a}}{3} + \frac{b \operatorname{arctanh}\left(\frac{2cx^3 + b}{2\sqrt{c}\sqrt{cx^6 + bx^3 + a}}\right)}{6\sqrt{c}} + \frac{\sqrt{cx^6 + bx^3 + a}}{3}$$

Result(type 8, 20 leaves):

$$\int \frac{\sqrt{cx^6 + bx^3 + a}}{x} dx$$

Problem 51: Unable to integrate problem.

$$\int x\sqrt{cx^6 + bx^3 + a} dx$$

Optimal(type 6, 116 leaves, 2 steps):

$$\frac{x^2 \operatorname{AppellF1}\left(\frac{2}{3}, -\frac{1}{2}, -\frac{1}{2}, \frac{5}{3}, -\frac{2cx^3}{b - \sqrt{-4ac + b^2}}, -\frac{2cx^3}{b + \sqrt{-4ac + b^2}}\right)\sqrt{cx^6 + bx^3 + a}}{2\sqrt{1 + \frac{2cx^3}{b - \sqrt{-4ac + b^2}}}\sqrt{1 + \frac{2cx^3}{b + \sqrt{-4ac + b^2}}}}$$

Result(type 8, 48 leaves):

$$\frac{x^2\sqrt{cx^6 + bx^3 + a}}{5} + \int \frac{3x(bx^3 + 2a)}{10\sqrt{cx^6 + bx^3 + a}} dx$$

Problem 52: Unable to integrate problem.

$$\int \frac{\sqrt{cx^6 + bx^3 + a}}{x^3} dx$$

Optimal(type 6, 116 leaves, 2 steps):

$$\frac{\text{AppellF1}\left(-\frac{2}{3}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{3}, -\frac{2cx^3}{b-\sqrt{-4ac+b^2}}, -\frac{2cx^3}{b+\sqrt{-4ac+b^2}}\right)\sqrt{cx^6+bx^3+a}}{2x^2\sqrt{1+\frac{2cx^3}{b-\sqrt{-4ac+b^2}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{-4ac+b^2}}}}$$

Result(type 8, 47 leaves):

$$-\frac{\sqrt{cx^6+bx^3+a}}{2x^2} + \int \frac{\frac{3cx^3}{2} + \frac{3b}{4}}{\sqrt{cx^6+bx^3+a}} dx$$

Problem 53: Unable to integrate problem.

$$\int x^2 (cx^6 + bx^3 + a)^{3/2} dx$$

Optimal(type 3, 106 leaves, 5 steps):

$$\frac{(2cx^3+b)(cx^6+bx^3+a)^{3/2}}{24c} + \frac{(-4ac+b^2)^2 \operatorname{arctanh}\left(\frac{2cx^3+b}{2\sqrt{c}\sqrt{cx^6+bx^3+a}}\right)}{128c^5/2} - \frac{(-4ac+b^2)(2cx^3+b)\sqrt{cx^6+bx^3+a}}{64c^2}$$

Result(type 8, 109 leaves):

$$\frac{(16x^9c^3 + 24bx^6c^2 + 40a^2c^2x^3 + 2b^2cx^3 + 20abc - 3b^3)\sqrt{cx^6+bx^3+a}}{192c^2} + \int \frac{3(16a^2c^2 - 8ab^2c + b^4)x^2}{128c^2\sqrt{cx^6+bx^3+a}} dx$$

Problem 54: Unable to integrate problem.

$$\int \frac{(cx^6+bx^3+a)^{3/2}}{x^4} dx$$

Optimal(type 3, 122 leaves, 8 steps):

$$-\frac{(cx^6+bx^3+a)^{3/2}}{3x^3} - \frac{b \operatorname{arctanh}\left(\frac{bx^3+2a}{2\sqrt{a}\sqrt{cx^6+bx^3+a}}\right)\sqrt{a}}{2} + \frac{(4ac+b^2) \operatorname{arctanh}\left(\frac{2cx^3+b}{2\sqrt{c}\sqrt{cx^6+bx^3+a}}\right)}{8\sqrt{c}} + \frac{(2cx^3+3b)\sqrt{cx^6+bx^3+a}}{4}$$

Result(type 8, 77 leaves):

$$-\frac{a\sqrt{cx^6+bx^3+a}}{3x^3} + \int \frac{2c^2x^9 + 4bcx^6 + 4acx^3 + 2b^2x^3 + 3ab}{2x\sqrt{cx^6+bx^3+a}} dx$$

Problem 55: Unable to integrate problem.

$$\int \frac{(cx^6 + bx^3 + a)^{3/2}}{x^7} dx$$

Optimal(type 3, 123 leaves, 8 steps):

$$-\frac{(cx^6 + bx^3 + a)^{3/2}}{6x^6} - \frac{(4ac + b^2) \operatorname{arctanh}\left(\frac{bx^3 + 2a}{2\sqrt{a}\sqrt{cx^6 + bx^3 + a}}\right)}{8\sqrt{a}} + \frac{b \operatorname{arctanh}\left(\frac{2cx^3 + b}{2\sqrt{c}\sqrt{cx^6 + bx^3 + a}}\right)\sqrt{c}}{2} - \frac{(-2cx^3 + b)\sqrt{cx^6 + bx^3 + a}}{4x^3}$$

Result(type 8, 76 leaves):

$$-\frac{\sqrt{cx^6 + bx^3 + a}(5bx^3 + 2a)}{12x^6} + \int \frac{8c^2x^6 + 16bcx^3 + 12ac + 3b^2}{8x\sqrt{cx^6 + bx^3 + a}} dx$$

Problem 56: Unable to integrate problem.

$$\int \frac{(cx^6 + bx^3 + a)^{3/2}}{x^2} dx$$

Optimal(type 6, 117 leaves, 2 steps):

$$-\frac{a \operatorname{AppellF1}\left(-\frac{1}{3}, -\frac{3}{2}, -\frac{3}{2}, \frac{2}{3}, -\frac{2cx^3}{b - \sqrt{-4ac + b^2}}, -\frac{2cx^3}{b + \sqrt{-4ac + b^2}}\right)\sqrt{cx^6 + bx^3 + a}}{x \sqrt{1 + \frac{2cx^3}{b - \sqrt{-4ac + b^2}}} \sqrt{1 + \frac{2cx^3}{b + \sqrt{-4ac + b^2}}}}$$

Result(type 8, 74 leaves):

$$-\frac{\sqrt{cx^6 + bx^3 + a}(-10cx^6 - 19bx^3 + 80a)}{80x} + \int \frac{27x(20acx^3 + b^2x^3 + 12ab)}{160\sqrt{cx^6 + bx^3 + a}} dx$$

Problem 57: Unable to integrate problem.

$$\int \frac{x^{11}}{\sqrt{cx^6 + bx^3 + a}} dx$$

Optimal(type 3, 103 leaves, 5 steps):

$$-\frac{b(-12ac + 5b^2) \operatorname{arctanh}\left(\frac{2cx^3 + b}{2\sqrt{c}\sqrt{cx^6 + bx^3 + a}}\right)}{48c^7/2} + \frac{x^6\sqrt{cx^6 + bx^3 + a}}{9c} + \frac{(-10bcx^3 - 16ac + 15b^2)\sqrt{cx^6 + bx^3 + a}}{72c^3}$$

Result(type 8, 80 leaves):

$$-\frac{(-8c^2x^6 + 10bcx^3 + 16ac - 15b^2)\sqrt{cx^6 + bx^3 + a}}{72c^3} + \int \frac{b(12ac - 5b^2)x^2}{16c^3\sqrt{cx^6 + bx^3 + a}} dx$$

Problem 58: Unable to integrate problem.

$$\int \frac{x^8}{\sqrt{cx^6 + bx^3 + a}} dx$$

Optimal(type 3, 86 leaves, 5 steps):

$$\frac{(-4ac + 3b^2) \operatorname{arctanh}\left(\frac{2cx^3 + b}{2\sqrt{c}\sqrt{cx^6 + bx^3 + a}}\right)}{24c^5/2} - \frac{b\sqrt{cx^6 + bx^3 + a}}{4c^2} + \frac{x^3\sqrt{cx^6 + bx^3 + a}}{6c}$$

Result(type 8, 64 leaves):

$$-\frac{(-2cx^3 + 3b)\sqrt{cx^6 + bx^3 + a}}{12c^2} + \int -\frac{x^2(4ac - 3b^2)}{8c^2\sqrt{cx^6 + bx^3 + a}} dx$$

Problem 59: Unable to integrate problem.

$$\int \frac{1}{x^4\sqrt{cx^6 + bx^3 + a}} dx$$

Optimal(type 3, 58 leaves, 4 steps):

$$\frac{b \operatorname{arctanh}\left(\frac{bx^3 + 2a}{2\sqrt{a}\sqrt{cx^6 + bx^3 + a}}\right)}{6a^3/2} - \frac{\sqrt{cx^6 + bx^3 + a}}{3ax^3}$$

Result(type 8, 48 leaves):

$$-\frac{\sqrt{cx^6 + bx^3 + a}}{3ax^3} + \int -\frac{b}{2ax\sqrt{cx^6 + bx^3 + a}} dx$$

Problem 60: Unable to integrate problem.

$$\int \frac{1}{x^{13}\sqrt{cx^6 + bx^3 + a}} dx$$

Optimal(type 3, 166 leaves, 7 steps):

$$\begin{aligned}
& - \frac{(48 a^2 c^2 - 120 a b^2 c + 35 b^4) \operatorname{arctanh}\left(\frac{b x^3 + 2 a}{2 \sqrt{a} \sqrt{c x^6 + b x^3 + a}}\right)}{384 a^9 / 2} - \frac{\sqrt{c x^6 + b x^3 + a}}{12 a x^{12}} + \frac{7 b \sqrt{c x^6 + b x^3 + a}}{72 a^2 x^9} - \frac{(-36 a c + 35 b^2) \sqrt{c x^6 + b x^3 + a}}{288 a^3 x^6} \\
& + \frac{5 b (-44 a c + 21 b^2) \sqrt{c x^6 + b x^3 + a}}{576 a^4 x^3}
\end{aligned}$$

Result (type 8, 117 leaves):

$$\frac{-\sqrt{c x^6 + b x^3 + a} (220 a b c x^9 - 105 b^3 x^9 - 72 a^2 c x^6 + 70 a b^2 x^6 - 56 a^2 b x^3 + 48 a^3)}{576 a^4 x^{12}} + \int \frac{48 a^2 c^2 - 120 a b^2 c + 35 b^4}{128 a^4 x \sqrt{c x^6 + b x^3 + a}} dx$$

Problem 61: Unable to integrate problem.

$$\int \frac{x}{\sqrt{c x^6 + b x^3 + a}} dx$$

Optimal (type 6, 116 leaves, 2 steps):

$$\frac{x^2 \operatorname{AppellF1}\left(\frac{2}{3}, \frac{1}{2}, \frac{1}{2}, \frac{5}{3}, -\frac{2 c x^3}{b - \sqrt{-4 a c + b^2}}, -\frac{2 c x^3}{b + \sqrt{-4 a c + b^2}}\right) \sqrt{1 + \frac{2 c x^3}{b - \sqrt{-4 a c + b^2}}} \sqrt{1 + \frac{2 c x^3}{b + \sqrt{-4 a c + b^2}}}}{2 \sqrt{c x^6 + b x^3 + a}}$$

Result (type 8, 18 leaves):

$$\int \frac{x}{\sqrt{c x^6 + b x^3 + a}} dx$$

Problem 62: Unable to integrate problem.

$$\int \frac{1}{\sqrt{c x^6 + b x^3 + a}} dx$$

Optimal (type 6, 113 leaves, 2 steps):

$$\frac{x \operatorname{AppellF1}\left(\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, -\frac{2 c x^3}{b - \sqrt{-4 a c + b^2}}, -\frac{2 c x^3}{b + \sqrt{-4 a c + b^2}}\right) \sqrt{1 + \frac{2 c x^3}{b - \sqrt{-4 a c + b^2}}} \sqrt{1 + \frac{2 c x^3}{b + \sqrt{-4 a c + b^2}}}}{\sqrt{c x^6 + b x^3 + a}}$$

Result (type 8, 16 leaves):

$$\int \frac{1}{\sqrt{c x^6 + b x^3 + a}} dx$$

Problem 63: Unable to integrate problem.

$$\int \frac{1}{x^3 \sqrt{cx^6 + bx^3 + a}} dx$$

Optimal(type 6, 116 leaves, 2 steps):

$$\frac{\text{AppellF1}\left(-\frac{2}{3}, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, -\frac{2cx^3}{b - \sqrt{-4ac + b^2}}, -\frac{2cx^3}{b + \sqrt{-4ac + b^2}}\right) \sqrt{1 + \frac{2cx^3}{b - \sqrt{-4ac + b^2}}} \sqrt{1 + \frac{2cx^3}{b + \sqrt{-4ac + b^2}}}}{2x^2 \sqrt{cx^6 + bx^3 + a}}$$

Result(type 8, 52 leaves):

$$-\frac{\sqrt{cx^6 + bx^3 + a}}{2ax^2} + \int -\frac{-2cx^3 + b}{4a\sqrt{cx^6 + bx^3 + a}} dx$$

Problem 65: Unable to integrate problem.

$$\int \frac{1}{x (cx^6 + bx^3 + a)^{3/2}} dx$$

Optimal(type 3, 78 leaves, 5 steps):

$$-\frac{\operatorname{arctanh}\left(\frac{bx^3 + 2a}{2\sqrt{a}\sqrt{cx^6 + bx^3 + a}}\right)}{3a^{3/2}} + \frac{2(bc x^3 - 2ac + b^2)}{3a(-4ac + b^2)\sqrt{cx^6 + bx^3 + a}}$$

Result(type 8, 20 leaves):

$$\int \frac{1}{x (cx^6 + bx^3 + a)^{3/2}} dx$$

Problem 66: Unable to integrate problem.

$$\int \frac{1}{x^{10} (cx^6 + bx^3 + a)^{3/2}} dx$$

Optimal(type 3, 230 leaves, 7 steps):

$$\frac{5b(-12ac + 7b^2) \operatorname{arctanh}\left(\frac{bx^3 + 2a}{2\sqrt{a}\sqrt{cx^6 + bx^3 + a}}\right)}{48a^9/2} + \frac{2(bc x^3 - 2ac + b^2)}{3a(-4ac + b^2)x^9\sqrt{cx^6 + bx^3 + a}} - \frac{(-16ac + 7b^2)\sqrt{cx^6 + bx^3 + a}}{9a^2(-4ac + b^2)x^9}$$

$$+ \frac{b(-116ac + 35b^2)\sqrt{cx^6 + bx^3 + a}}{36a^3(-4ac + b^2)x^6} - \frac{(256a^2c^2 - 460ab^2c + 105b^4)\sqrt{cx^6 + bx^3 + a}}{72a^4(-4ac + b^2)x^3}$$

Result(type 8, 159 leaves):

$$-\frac{\sqrt{cx^6 + bx^3 + a} (-40acx^6 + 57b^2x^6 - 22abx^3 + 8a^2)}{72a^4x^9} + \int \frac{28ab^2cx^6 - 19b^3cx^6 + 16a^2c^2x^3 + 12ab^2cx^3 - 19b^4x^3 + 60a^2bc - 35ab^3}{16a^4xc \left(x^6 + \frac{bx^3}{c} + \frac{a}{c}\right) \sqrt{cx^6 + bx^3 + a}} dx$$

Problem 67: Unable to integrate problem.

$$\int \frac{x^3}{(cx^6 + bx^3 + a)^{3/2}} dx$$

Optimal(type 6, 119 leaves, 2 steps):

$$\frac{x^4 \operatorname{AppellF1}\left(\frac{4}{3}, \frac{3}{2}, \frac{3}{2}, \frac{7}{3}, -\frac{2cx^3}{b - \sqrt{-4ac + b^2}}, -\frac{2cx^3}{b + \sqrt{-4ac + b^2}}\right) \sqrt{1 + \frac{2cx^3}{b - \sqrt{-4ac + b^2}}} \sqrt{1 + \frac{2cx^3}{b + \sqrt{-4ac + b^2}}}}{4a\sqrt{cx^6 + bx^3 + a}}$$

Result(type 8, 20 leaves):

$$\int \frac{x^3}{(cx^6 + bx^3 + a)^{3/2}} dx$$

Problem 68: Unable to integrate problem.

$$\int \frac{1}{x^3 (cx^6 + bx^3 + a)^{3/2}} dx$$

Optimal(type 6, 119 leaves, 2 steps):

$$\frac{\operatorname{AppellF1}\left(-\frac{2}{3}, \frac{3}{2}, \frac{3}{2}, \frac{1}{3}, -\frac{2cx^3}{b - \sqrt{-4ac + b^2}}, -\frac{2cx^3}{b + \sqrt{-4ac + b^2}}\right) \sqrt{1 + \frac{2cx^3}{b - \sqrt{-4ac + b^2}}} \sqrt{1 + \frac{2cx^3}{b + \sqrt{-4ac + b^2}}}}{2ax^2\sqrt{cx^6 + bx^3 + a}}$$

Result(type 8, 100 leaves):

$$-\frac{\sqrt{cx^6 + bx^3 + a}}{2a^2x^2} + \int \frac{-2c^2x^9 - b^2cx^6 + 2acx^3 + b^2x^3 + 5ab}{4a^2c \left(x^6 + \frac{bx^3}{c} + \frac{a}{c}\right) \sqrt{cx^6 + bx^3 + a}} dx$$

Problem 69: Unable to integrate problem.

$$\int \frac{(dx)^m}{cx^6 + bx^3 + a} dx$$

Optimal(type 5, 157 leaves, 3 steps):

$$\frac{2c(dx)^{1+m} \operatorname{hypergeom}\left(\left[1, \frac{1}{3} + \frac{m}{3}\right], \left[\frac{4}{3} + \frac{m}{3}\right], -\frac{2cx^3}{b - \sqrt{-4ac + b^2}}\right)}{d(1+m)(b - \sqrt{-4ac + b^2})\sqrt{-4ac + b^2}} - \frac{2c(dx)^{1+m} \operatorname{hypergeom}\left(\left[1, \frac{1}{3} + \frac{m}{3}\right], \left[\frac{4}{3} + \frac{m}{3}\right], -\frac{2cx^3}{b + \sqrt{-4ac + b^2}}\right)}{d(1+m)\sqrt{-4ac + b^2}(b + \sqrt{-4ac + b^2})}$$

Result(type 8, 22 leaves):

$$\int \frac{(dx)^m}{cx^6 + bx^3 + a} dx$$

Problem 70: Unable to integrate problem.

$$\int x^5 (cx^6 + bx^3 + a)^p dx$$

Optimal(type 5, 146 leaves, 3 steps):

$$\frac{(cx^6 + bx^3 + a)^{1+p}}{6c(1+p)} + \frac{2^p b (cx^6 + bx^3 + a)^{1+p} \operatorname{hypergeom}\left(\left[-p, 1+p\right], \left[2+p\right], \frac{2cx^3 + \sqrt{-4ac + b^2} + b}{2\sqrt{-4ac + b^2}}\right) \left(\frac{-2cx^3 + \sqrt{-4ac + b^2} - b}{\sqrt{-4ac + b^2}}\right)^{-1-p}}{3c(1+p)\sqrt{-4ac + b^2}}$$

Result(type 8, 20 leaves):

$$\int x^5 (cx^6 + bx^3 + a)^p dx$$

Problem 71: Unable to integrate problem.

$$\int x^2 (cx^6 + bx^3 + a)^p dx$$

Optimal(type 5, 117 leaves, 2 steps):

$$\frac{2^{1+p} (cx^6 + bx^3 + a)^{1+p} \operatorname{hypergeom}\left(\left[-p, 1+p\right], \left[2+p\right], \frac{2cx^3 + \sqrt{-4ac + b^2} + b}{2\sqrt{-4ac + b^2}}\right) \left(\frac{-2cx^3 + \sqrt{-4ac + b^2} - b}{\sqrt{-4ac + b^2}}\right)^{-1-p}}{3(1+p)\sqrt{-4ac + b^2}}$$

Result(type 8, 20 leaves):

$$\int x^2 (cx^6 + bx^3 + a)^p dx$$

Problem 72: Unable to integrate problem.

$$\int \frac{(cx^6 + bx^3 + a)^p}{x} dx$$

Optimal(type 6, 147 leaves, 3 steps):

$$\frac{2^{-1+2p} (cx^6 + bx^3 + a)^p \operatorname{AppellF1}\left(-2p, -p, -p, 1 - 2p, \frac{-b - \sqrt{-4ac + b^2}}{2cx^3}, \frac{-b + \sqrt{-4ac + b^2}}{2cx^3}\right)}{3p \left(\frac{2cx^3 - \sqrt{-4ac + b^2} + b}{cx^3}\right)^p \left(\frac{2cx^3 + \sqrt{-4ac + b^2} + b}{cx^3}\right)^p}$$

Result(type 8, 20 leaves):

$$\int \frac{(cx^6 + bx^3 + a)^p}{x} dx$$

Problem 73: Unable to integrate problem.

$$\int \frac{(cx^6 + bx^3 + a)^p}{x^4} dx$$

Optimal(type 6, 152 leaves, 3 steps):

$$\frac{4^p (cx^6 + bx^3 + a)^p \operatorname{AppellF1}\left(1 - 2p, -p, -p, 2 - 2p, \frac{-b - \sqrt{-4ac + b^2}}{2cx^3}, \frac{-b + \sqrt{-4ac + b^2}}{2cx^3}\right)}{3(1 - 2p)x^3 \left(\frac{2cx^3 - \sqrt{-4ac + b^2} + b}{cx^3}\right)^p \left(\frac{2cx^3 + \sqrt{-4ac + b^2} + b}{cx^3}\right)^p}$$

Result(type 8, 20 leaves):

$$\int \frac{(cx^6 + bx^3 + a)^p}{x^4} dx$$

Problem 82: Unable to integrate problem.

$$\int \frac{x^m}{cx^8 + bx^4 + a} dx$$

Optimal(type 5, 147 leaves, 3 steps):

$$\frac{2cx^{1+m} \operatorname{hypergeom}\left(\left[1, \frac{1}{4} + \frac{m}{4}\right], \left[\frac{5}{4} + \frac{m}{4}\right], -\frac{2cx^4}{b - \sqrt{-4ac + b^2}}\right)}{(1+m)(b - \sqrt{-4ac + b^2})\sqrt{-4ac + b^2}} - \frac{2cx^{1+m} \operatorname{hypergeom}\left(\left[1, \frac{1}{4} + \frac{m}{4}\right], \left[\frac{5}{4} + \frac{m}{4}\right], -\frac{2cx^4}{b + \sqrt{-4ac + b^2}}\right)}{(1+m)\sqrt{-4ac + b^2}(b + \sqrt{-4ac + b^2})}$$

Result(type 8, 20 leaves):

$$\int \frac{x^m}{cx^8 + bx^4 + a} dx$$

Problem 85: Unable to integrate problem.

$$\int \frac{x^m}{x^8 + x^4 + 1} dx$$

Optimal(type 5, 107 leaves, 3 steps):

$$-\frac{2x^{1+m} \operatorname{hypergeom}\left(\left[1, \frac{1}{4} + \frac{m}{4}\right], \left[\frac{5}{4} + \frac{m}{4}\right], -\frac{2x^4}{1+I\sqrt{3}}\right) \sqrt{3}}{3(1+m)(1-\sqrt{3})} + \frac{2x^{1+m} \operatorname{hypergeom}\left(\left[1, \frac{1}{4} + \frac{m}{4}\right], \left[\frac{5}{4} + \frac{m}{4}\right], -\frac{2x^4}{1-I\sqrt{3}}\right) \sqrt{3}}{3(1+m)(1+\sqrt{3})}$$

Result(type 8, 16 leaves):

$$\int \frac{x^m}{x^8 + x^4 + 1} dx$$

Problem 94: Unable to integrate problem.

$$\int \frac{x^m}{x^8 - x^4 + 1} dx$$

Optimal(type 5, 107 leaves, 3 steps):

$$-\frac{2x^{1+m} \operatorname{hypergeom}\left(\left[1, \frac{1}{4} + \frac{m}{4}\right], \left[\frac{5}{4} + \frac{m}{4}\right], \frac{2x^4}{1+I\sqrt{3}}\right) \sqrt{3}}{3(1+m)(1-\sqrt{3})} + \frac{2x^{1+m} \operatorname{hypergeom}\left(\left[1, \frac{1}{4} + \frac{m}{4}\right], \left[\frac{5}{4} + \frac{m}{4}\right], \frac{2x^4}{1-I\sqrt{3}}\right) \sqrt{3}}{3(1+m)(1+\sqrt{3})}$$

Result(type 8, 18 leaves):

$$\int \frac{x^m}{x^8 - x^4 + 1} dx$$

Problem 98: Result is not expressed in closed-form.

$$\int \frac{1}{x^2(x^8 - x^4 + 1)} dx$$

Optimal(type 3, 312 leaves, 22 steps):

$$-\frac{1}{x} + \frac{\ln\left(1+x^2-x\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\right)\left(\frac{\sqrt{2}}{2}-\frac{\sqrt{6}}{6}\right)}{8} - \frac{\ln\left(1+x^2+x\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\right)\left(\frac{\sqrt{2}}{2}-\frac{\sqrt{6}}{6}\right)}{8} + \frac{\arctan\left(\frac{-2x+\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}}{\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}}\right)}{4\left(\frac{3\sqrt{2}}{2}-\frac{\sqrt{6}}{2}\right)}$$

$$\begin{aligned}
& - \frac{\arctan\left(\frac{2x + \frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}}{\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}}\right)}{4\left(\frac{3\sqrt{2}}{2} - \frac{\sqrt{6}}{2}\right)} - \frac{\ln\left(1 + x^2 - x\left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right)\right)\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{6}}{6}\right)}{8} + \frac{\ln\left(1 + x^2 + x\left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right)\right)\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{6}}{6}\right)}{8} \\
& - \frac{\arctan\left(\frac{-2x + \frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}}{\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}}\right)}{4\left(\frac{3\sqrt{2}}{2} + \frac{\sqrt{6}}{2}\right)} + \frac{\arctan\left(\frac{2x + \frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}}{\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}}\right)}{4\left(\frac{3\sqrt{2}}{2} + \frac{\sqrt{6}}{2}\right)}
\end{aligned}$$

Result(type 7, 51 leaves):

$$- \frac{\left(\sum_{R=\text{RootOf}(Z^8 - Z^4 + 1)} \frac{(R^6 - R^2) \ln(x - R)}{2R^7 - R^3}\right)}{4} - \frac{1}{x}$$

Problem 99: Result is not expressed in closed-form.

$$\int \frac{1}{x^4 (x^8 - x^4 + 1)} dx$$

Optimal(type 3, 304 leaves, 20 steps):

$$\begin{aligned}
& - \frac{1}{3x^3} + \frac{\arctan\left(\frac{-2x + \frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}}{\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}}\right)\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{6}}{6}\right)}{4} - \frac{\arctan\left(\frac{2x + \frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}}{\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}}\right)\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{6}}{6}\right)}{4} \\
& + \frac{\ln\left(1 + x^2 - x\left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right)\right)\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{6}}{6}\right)}{8} - \frac{\ln\left(1 + x^2 + x\left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right)\right)\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{6}}{6}\right)}{8} \\
& - \frac{\arctan\left(\frac{-2x + \frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}}{\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}}\right)\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{6}}{6}\right)}{4} + \frac{\arctan\left(\frac{2x + \frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}}{\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}}\right)\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{6}}{6}\right)}{4}
\end{aligned}$$

$$-\frac{\ln\left(1+x^2-x\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)\right)\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{6}}{6}\right)}{8}+\frac{\ln\left(1+x^2+x\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)\right)\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{6}}{6}\right)}{8}$$

Result(type 7, 49 leaves):

$$-\frac{1}{3x^3}+\frac{\left(\sum_{R=\text{RootOf}(Z^8-Z^4+1)}\frac{(-R^4+1)\ln(x-R)}{2R^7-R^3}\right)}{4}$$

Problem 100: Result more than twice size of optimal antiderivative.

$$\int\frac{x^9}{x^8+3x^4+1}dx$$

Optimal(type 3, 51 leaves, 5 steps):

$$\frac{x^2}{2}+\frac{\arctan\left(x^2\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\right)\left(1-\frac{2\sqrt{5}}{5}\right)}{2}-\frac{\arctan\left(\frac{x^2\sqrt{2}}{\sqrt{3+\sqrt{5}}}\right)\left(1+\frac{2\sqrt{5}}{5}\right)}{2}$$

Result(type 3, 116 leaves):

$$\frac{x^2}{2}-\frac{7\sqrt{5}\arctan\left(\frac{4x^2}{2\sqrt{5}+2}\right)}{5(2\sqrt{5}+2)}-\frac{3\arctan\left(\frac{4x^2}{2\sqrt{5}+2}\right)}{2\sqrt{5}+2}+\frac{7\sqrt{5}\arctan\left(\frac{4x^2}{2\sqrt{5}-2}\right)}{5(2\sqrt{5}-2)}-\frac{3\arctan\left(\frac{4x^2}{2\sqrt{5}-2}\right)}{2\sqrt{5}-2}$$

Problem 101: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{x^3(x^8+3x^4+1)}dx$$

Optimal(type 3, 54 leaves, 5 steps):

$$-\frac{1}{2x^2}-\frac{\arctan\left(x^2\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\right)(3+\sqrt{5})^{3/2}\sqrt{10}}{40}+\frac{\arctan\left(\frac{x^2\sqrt{2}}{\sqrt{3+\sqrt{5}}}\right)\left(1-\frac{2\sqrt{5}}{5}\right)}{2}$$

Result(type 3, 116 leaves):

$$-\frac{1}{2x^2}+\frac{3\sqrt{5}\arctan\left(\frac{4x^2}{2\sqrt{5}+2}\right)}{5(2\sqrt{5}+2)}-\frac{\arctan\left(\frac{4x^2}{2\sqrt{5}+2}\right)}{2\sqrt{5}+2}-\frac{\arctan\left(\frac{4x^2}{2\sqrt{5}-2}\right)}{2\sqrt{5}-2}-\frac{3\sqrt{5}\arctan\left(\frac{4x^2}{2\sqrt{5}-2}\right)}{5(2\sqrt{5}-2)}$$

Problem 102: Result is not expressed in closed-form.

$$\int \frac{x^4}{x^8 + 3x^4 + 1} dx$$

Optimal (type 3, 293 leaves, 19 steps):

$$\begin{aligned} & \frac{\arctan\left(-1 + \frac{2^{3/4}x}{(3-\sqrt{5})^{1/4}}\right) (3-\sqrt{5})^{1/4} 2^{1/4}\sqrt{5}}{20} - \frac{\arctan\left(1 + \frac{2^{3/4}x}{(3-\sqrt{5})^{1/4}}\right) (3-\sqrt{5})^{1/4} 2^{1/4}\sqrt{5}}{20} \\ & + \frac{\ln\left(2x^2 - 2 \cdot 2^{1/4}x(3-\sqrt{5})^{1/4} + \sqrt{5} - 1\right) (3-\sqrt{5})^{1/4} 2^{1/4}\sqrt{5}}{40} - \frac{\ln\left(2x^2 + 2 \cdot 2^{1/4}x(3-\sqrt{5})^{1/4} + \sqrt{5} - 1\right) (3-\sqrt{5})^{1/4} 2^{1/4}\sqrt{5}}{40} \\ & + \frac{\arctan\left(-1 + \frac{2^{3/4}x}{(3+\sqrt{5})^{1/4}}\right) (3+\sqrt{5})^{1/4} 2^{1/4}\sqrt{5}}{20} + \frac{\arctan\left(1 + \frac{2^{3/4}x}{(3+\sqrt{5})^{1/4}}\right) (3+\sqrt{5})^{1/4} 2^{1/4}\sqrt{5}}{20} \\ & - \frac{\ln\left(2x^2 - 2 \cdot 2^{1/4}x(3+\sqrt{5})^{1/4} + \sqrt{5} + 1\right) (3+\sqrt{5})^{1/4} 2^{1/4}\sqrt{5}}{40} + \frac{\ln\left(2x^2 + 2 \cdot 2^{1/4}x(3+\sqrt{5})^{1/4} + \sqrt{5} + 1\right) (3+\sqrt{5})^{1/4} 2^{1/4}\sqrt{5}}{40} \end{aligned}$$

Result (type 7, 39 leaves):

$$\left(\frac{\sum_{R=\text{RootOf}(Z^8+3Z^4+1)} \frac{R^4 \ln(x-R)}{2R^7+3R^3}}{4} \right)$$

Problem 103: Result is not expressed in closed-form.

$$\int \frac{1}{x^8 + 3x^4 + 1} dx$$

Optimal (type 3, 221 leaves, 19 steps):

$$\begin{aligned} & \frac{\arctan\left(x\sqrt{\sqrt{5}-1}-1\right)\sqrt{-20+10\sqrt{5}}}{20} - \frac{\arctan\left(1+x\sqrt{\sqrt{5}-1}\right)\sqrt{-20+10\sqrt{5}}}{20} + \frac{\ln\left(1+2x^2+\sqrt{5}-2x\sqrt{\sqrt{5}+1}\right)\sqrt{-20+10\sqrt{5}}}{40} \\ & - \frac{\ln\left(1+2x^2+\sqrt{5}+2x\sqrt{\sqrt{5}+1}\right)\sqrt{-20+10\sqrt{5}}}{40} + \frac{\arctan\left(x\sqrt{\sqrt{5}+1}-1\right)\sqrt{20+10\sqrt{5}}}{20} + \frac{\arctan\left(1+x\sqrt{\sqrt{5}+1}\right)\sqrt{20+10\sqrt{5}}}{20} \\ & - \frac{\ln\left(-1+2x^2+\sqrt{5}-2x\sqrt{\sqrt{5}-1}\right)\sqrt{20+10\sqrt{5}}}{40} + \frac{\ln\left(-1+2x^2+\sqrt{5}+2x\sqrt{\sqrt{5}-1}\right)\sqrt{20+10\sqrt{5}}}{40} \end{aligned}$$

Result (type 7, 36 leaves):

$$\left(\frac{\sum_{R=\text{RootOf}(Z^8+3Z^4+1)} \frac{\ln(x-R)}{2R^7+3R^3}}{4} \right)$$

Problem 104: Result is not expressed in closed-form.

$$\int \frac{1}{x^2 (x^8 + 3x^4 + 1)} dx$$

Optimal (type 3, 270 leaves, 20 steps):

$$\begin{aligned} & -\frac{1}{x} + \frac{\arctan\left(-1 + \frac{2^{3/4}x}{(3+\sqrt{5})^{1/4}}\right) (6150 - 2750\sqrt{5})^{1/4}}{20} + \frac{\arctan\left(1 + \frac{2^{3/4}x}{(3+\sqrt{5})^{1/4}}\right) (6150 - 2750\sqrt{5})^{1/4}}{20} \\ & + \frac{\ln(2x^2 - 2 \cdot 2^{1/4}x(3+\sqrt{5})^{1/4} + \sqrt{5} + 1) (6150 - 2750\sqrt{5})^{1/4}}{40} - \frac{\ln(2x^2 + 2 \cdot 2^{1/4}x(3+\sqrt{5})^{1/4} + \sqrt{5} + 1) (6150 - 2750\sqrt{5})^{1/4}}{40} \\ & - \frac{\arctan\left(-1 + \frac{2^{3/4}x}{(3-\sqrt{5})^{1/4}}\right) (246 + 110\sqrt{5})^{1/4} \sqrt{5}}{20} - \frac{\arctan\left(1 + \frac{2^{3/4}x}{(3-\sqrt{5})^{1/4}}\right) (246 + 110\sqrt{5})^{1/4} \sqrt{5}}{20} \\ & - \frac{\ln(2x^2 - 2 \cdot 2^{1/4}x(3-\sqrt{5})^{1/4} + \sqrt{5} - 1) (246 + 110\sqrt{5})^{1/4} \sqrt{5}}{40} + \frac{\ln(2x^2 + 2 \cdot 2^{1/4}x(3-\sqrt{5})^{1/4} + \sqrt{5} - 1) (246 + 110\sqrt{5})^{1/4} \sqrt{5}}{40} \end{aligned}$$

Result (type 7, 51 leaves):

$$-\frac{\left(\sum_{R=\text{RootOf}(Z^8+3Z^4+1)} \frac{(R^6+3R^2)\ln(x-R)}{2R^7+3R^3}\right)}{4} - \frac{1}{x}$$

Problem 112: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^2} dx$$

Optimal (type 3, 188 leaves, 8 steps):

$$\begin{aligned} & -\frac{b(-11ac+3b^2)x}{c^3(-4ac+b^2)} + \frac{(-8ac+3b^2)x^2}{2c^2(-4ac+b^2)} - \frac{bx^3}{c(-4ac+b^2)} + \frac{x^4(bx+2a)}{(-4ac+b^2)(cx^2+bx+a)} + \frac{b(30a^2c^2-20ab^2c+3b^4)\operatorname{arctanh}\left(\frac{2cx+b}{\sqrt{-4ac+b^2}}\right)}{c^4(-4ac+b^2)^{3/2}} \\ & + \frac{(-2ac+3b^2)\ln(cx^2+bx+a)}{2c^4} \end{aligned}$$

Result (type 3, 661 leaves):

$$\begin{aligned} & \frac{x^2}{2c^2} - \frac{2bx}{c^3} - \frac{5bxa^2}{c^2(cx^2+bx+a)(4ac-b^2)} + \frac{5b^3xa}{c^3(cx^2+bx+a)(4ac-b^2)} - \frac{b^5x}{c^4(cx^2+bx+a)(4ac-b^2)} - \frac{2a^3}{c^2(cx^2+bx+a)(4ac-b^2)} \\ & + \frac{4a^2b^2}{c^3(cx^2+bx+a)(4ac-b^2)} - \frac{ab^4}{c^4(cx^2+bx+a)(4ac-b^2)} - \frac{4\ln((4ac-b^2)(cx^2+bx+a))a^2}{c^2(4ac-b^2)} \end{aligned}$$

$$\begin{aligned}
& + \frac{7 \ln((4ac - b^2)(cx^2 + bx + a)) ab^2}{c^3(4ac - b^2)} - \frac{3 \ln((4ac - b^2)(cx^2 + bx + a)) b^4}{2c^4(4ac - b^2)} + \frac{30 \arctan\left(\frac{2c(4ac - b^2)x + b(4ac - b^2)}{\sqrt{64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6}}\right) a^2 b}{c^2 \sqrt{64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6}} \\
& - \frac{20 \arctan\left(\frac{2c(4ac - b^2)x + b(4ac - b^2)}{\sqrt{64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6}}\right) ab^3}{c^3 \sqrt{64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6}} + \frac{3 \arctan\left(\frac{2c(4ac - b^2)x + b(4ac - b^2)}{\sqrt{64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6}}\right) b^5}{c^4 \sqrt{64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6}}
\end{aligned}$$

Problem 114: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^2 x^7} dx$$

Optimal (type 3, 194 leaves, 8 steps):

$$\begin{aligned}
& \frac{8ac - 3b^2}{2a^2(-4ac + b^2)x^2} + \frac{b(-11ac + 3b^2)}{a^3(-4ac + b^2)x} + \frac{bcx - 2ac + b^2}{a(-4ac + b^2)x^2(cx^2 + bx + a)} + \frac{b(30a^2c^2 - 20ab^2c + 3b^4) \operatorname{arctanh}\left(\frac{2cx + b}{\sqrt{-4ac + b^2}}\right)}{a^4(-4ac + b^2)^3/2} \\
& + \frac{(-2ac + 3b^2) \ln(x)}{a^4} - \frac{(-2ac + 3b^2) \ln(cx^2 + bx + a)}{2a^4}
\end{aligned}$$

Result (type 3, 645 leaves):

$$\begin{aligned}
& - \frac{1}{2a^2x^2} - \frac{2 \ln(x) c}{a^3} + \frac{3 \ln(x) b^2}{a^4} + \frac{2b}{a^3x} + \frac{3c^2bx}{a^2(cx^2 + bx + a)(4ac - b^2)} - \frac{cb^3x}{a^3(cx^2 + bx + a)(4ac - b^2)} - \frac{2c^2}{a(cx^2 + bx + a)(4ac - b^2)} \\
& + \frac{4b^2c}{a^2(cx^2 + bx + a)(4ac - b^2)} - \frac{b^4}{a^3(cx^2 + bx + a)(4ac - b^2)} + \frac{4c^2 \ln((4ac - b^2)(cx^2 + bx + a))}{a^2(4ac - b^2)} \\
& - \frac{7c \ln((4ac - b^2)(cx^2 + bx + a)) b^2}{a^3(4ac - b^2)} + \frac{3 \ln((4ac - b^2)(cx^2 + bx + a)) b^4}{2a^4(4ac - b^2)} + \frac{30 \arctan\left(\frac{2c(4ac - b^2)x + b(4ac - b^2)}{\sqrt{64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6}}\right) c^2 b}{a^2 \sqrt{64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6}} \\
& - \frac{20 \arctan\left(\frac{2c(4ac - b^2)x + b(4ac - b^2)}{\sqrt{64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6}}\right) cb^3}{a^3 \sqrt{64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6}} + \frac{3 \arctan\left(\frac{2c(4ac - b^2)x + b(4ac - b^2)}{\sqrt{64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6}}\right) b^5}{a^4 \sqrt{64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6}}
\end{aligned}$$

Problem 115: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^3 x} dx$$

Optimal (type 3, 180 leaves, 8 steps):

$$\begin{aligned} & -\frac{b(-7ac+b^2)x}{c^2(-4ac+b^2)^2} + \frac{x^4(bx+2a)}{2(-4ac+b^2)(cx^2+bx+a)^2} + \frac{x^2(a(-16ac+b^2)+b(-10ac+b^2)x)}{2c(-4ac+b^2)^2(cx^2+bx+a)} \\ & + \frac{b(30a^2c^2-10ab^2c+b^4)\operatorname{arctanh}\left(\frac{2cx+b}{\sqrt{-4ac+b^2}}\right)}{c^3(-4ac+b^2)^{5/2}} + \frac{\ln(cx^2+bx+a)}{2c^3} \end{aligned}$$

Result (type 3, 805 leaves):

$$\begin{aligned} & \frac{b(25a^2c^2-15ab^2c+2b^4)x^3}{c^2(16a^2c^2-8ab^2c+b^4)} + \frac{(32a^3c^3+11a^2b^2c^2-19ab^4c+3b^6)x^2}{2c^3(16a^2c^2-8ab^2c+b^4)} + \frac{ab(31a^2c^2-22ab^2c+3b^4)x}{(16a^2c^2-8ab^2c+b^4)c^3} + \frac{3a^2(8a^2c^2-7ab^2c+b^4)}{2c^3(16a^2c^2-8ab^2c+b^4)} \\ & + \frac{\ln(c^2(16a^2c^2-8ab^2c+b^4)(cx^2+bx+a))}{2c^3} - \frac{30\operatorname{arctan}\left(\frac{2c^3(16a^2c^2-8ab^2c+b^4)x+c^2(16a^2c^2-8ab^2c+b^4)b}{\sqrt{1024a^5c^9-1280a^4b^2c^8+640a^3b^4c^7-160a^2b^6c^6+20ab^8c^5-b^{10}c^4}}\right)a^2bc}{\sqrt{1024a^5c^9-1280a^4b^2c^8+640a^3b^4c^7-160a^2b^6c^6+20ab^8c^5-b^{10}c^4}} \\ & + \frac{10\operatorname{arctan}\left(\frac{2c^3(16a^2c^2-8ab^2c+b^4)x+c^2(16a^2c^2-8ab^2c+b^4)b}{\sqrt{1024a^5c^9-1280a^4b^2c^8+640a^3b^4c^7-160a^2b^6c^6+20ab^8c^5-b^{10}c^4}}\right)ab^3}{\sqrt{1024a^5c^9-1280a^4b^2c^8+640a^3b^4c^7-160a^2b^6c^6+20ab^8c^5-b^{10}c^4}} \\ & - \frac{\operatorname{arctan}\left(\frac{2c^3(16a^2c^2-8ab^2c+b^4)x+c^2(16a^2c^2-8ab^2c+b^4)b}{\sqrt{1024a^5c^9-1280a^4b^2c^8+640a^3b^4c^7-160a^2b^6c^6+20ab^8c^5-b^{10}c^4}}\right)b^5}{\sqrt{1024a^5c^9-1280a^4b^2c^8+640a^3b^4c^7-160a^2b^6c^6+20ab^8c^5-b^{10}c^4}c} \end{aligned}$$

Problem 116: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^3 x^3} dx$$

Optimal (type 3, 99 leaves, 5 steps):

$$-\frac{x^3(2cx+b)}{2(-4ac+b^2)(cx^2+bx+a)^2} + \frac{3bx(bx+2a)}{2(-4ac+b^2)^2(cx^2+bx+a)} + \frac{6ab\operatorname{arctanh}\left(\frac{2cx+b}{\sqrt{-4ac+b^2}}\right)}{(-4ac+b^2)^{5/2}}$$

Result (type 3, 222 leaves):

$$\begin{aligned}
& - \frac{3abcx^3}{16a^2c^2 - 8ab^2c + b^4} - \frac{(16a^2c^2 + ab^2c + b^4)x^2}{2c(16a^2c^2 - 8ab^2c + b^4)} - \frac{(5ac + b^2)abx}{c(16a^2c^2 - 8ab^2c + b^4)} - \frac{a^2(8ac + b^2)}{2c(16a^2c^2 - 8ab^2c + b^4)} \\
& \qquad \qquad \qquad (cx^2 + bx + a)^2 \\
& - \frac{6ab \arctan\left(\frac{2cx + b}{\sqrt{4ac - b^2}}\right)}{(16a^2c^2 - 8ab^2c + b^4)\sqrt{4ac - b^2}}
\end{aligned}$$

Problem 119: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^3 x^8} dx$$

Optimal (type 3, 229 leaves, 9 steps):

$$\begin{aligned}
& - \frac{3(-5ac + b^2)(-2ac + b^2)}{a^3(-4ac + b^2)^2 x} + \frac{bcx - 2ac + b^2}{2a(-4ac + b^2)x(cx^2 + bx + a)^2} + \frac{3b^4 - 20ab^2c + 20a^2c^2 + 3bc(-6ac + b^2)x}{2a^2(-4ac + b^2)^2 x(cx^2 + bx + a)} \\
& - \frac{3(-20a^3c^3 + 30a^2b^2c^2 - 10ab^4c + b^6) \operatorname{arctanh}\left(\frac{2cx + b}{\sqrt{-4ac + b^2}}\right)}{a^4(-4ac + b^2)^{5/2}} - \frac{3b \ln(x)}{a^4} + \frac{3b \ln(cx^2 + bx + a)}{2a^4}
\end{aligned}$$

Result (type 3, 1437 leaves):

$$\begin{aligned}
& - \frac{1}{a^3 x} - \frac{3b \ln(x)}{a^4} - \frac{14c^4 x^3}{a(cx^2 + bx + a)^2(16a^2c^2 - 8ab^2c + b^4)} + \frac{13c^3 x^3 b^2}{a^2(cx^2 + bx + a)^2(16a^2c^2 - 8ab^2c + b^4)} \\
& - \frac{2c^2 x^3 b^4}{a^3(cx^2 + bx + a)^2(16a^2c^2 - 8ab^2c + b^4)} - \frac{37c^3 b x^2}{a(cx^2 + bx + a)^2(16a^2c^2 - 8ab^2c + b^4)} + \frac{55c^2 b^3 x^2}{2a^2(cx^2 + bx + a)^2(16a^2c^2 - 8ab^2c + b^4)} \\
& - \frac{4cb^5 x^2}{a^3(cx^2 + bx + a)^2(16a^2c^2 - 8ab^2c + b^4)} - \frac{18x c^3}{(cx^2 + bx + a)^2(16a^2c^2 - 8ab^2c + b^4)} - \frac{7xb^2 c^2}{a(cx^2 + bx + a)^2(16a^2c^2 - 8ab^2c + b^4)} \\
& + \frac{12xb^4 c}{a^2(cx^2 + bx + a)^2(16a^2c^2 - 8ab^2c + b^4)} - \frac{2xb^6}{a^3(cx^2 + bx + a)^2(16a^2c^2 - 8ab^2c + b^4)} - \frac{29b c^2}{(cx^2 + bx + a)^2(16a^2c^2 - 8ab^2c + b^4)} \\
& + \frac{18b^3 c}{a(cx^2 + bx + a)^2(16a^2c^2 - 8ab^2c + b^4)} - \frac{5b^5}{2a^2(cx^2 + bx + a)^2(16a^2c^2 - 8ab^2c + b^4)} \\
& + \frac{24c^2 \ln((16a^2c^2 - 8ab^2c + b^4)(cx^2 + bx + a)) b}{a^2(16a^2c^2 - 8ab^2c + b^4)} - \frac{12c \ln((16a^2c^2 - 8ab^2c + b^4)(cx^2 + bx + a)) b^3}{a^3(16a^2c^2 - 8ab^2c + b^4)} \\
& + \frac{3 \ln((16a^2c^2 - 8ab^2c + b^4)(cx^2 + bx + a)) b^5}{2a^4(16a^2c^2 - 8ab^2c + b^4)} - \frac{60 \arctan\left(\frac{2c(16a^2c^2 - 8ab^2c + b^4)x + b(16a^2c^2 - 8ab^2c + b^4)}{\sqrt{1024c^5 a^5 - 1280c^4 b^2 a^4 + 640c^3 b^4 a^3 - 160c^2 b^6 a^2 + 20cb^8 a - b^{10}}}\right) c^3}{a\sqrt{1024c^5 a^5 - 1280c^4 b^2 a^4 + 640c^3 b^4 a^3 - 160c^2 b^6 a^2 + 20cb^8 a - b^{10}}}
\end{aligned}$$

$$\begin{aligned}
& + \frac{90 \arctan\left(\frac{2c(16a^2c^2 - 8ab^2c + b^4)x + b(16a^2c^2 - 8ab^2c + b^4)}{\sqrt{1024c^5a^5 - 1280c^4b^2a^4 + 640c^3b^4a^3 - 160c^2b^6a^2 + 20cb^8a - b^{10}}}\right) b^2 c^2}{a^2 \sqrt{1024c^5a^5 - 1280c^4b^2a^4 + 640c^3b^4a^3 - 160c^2b^6a^2 + 20cb^8a - b^{10}}} \\
& - \frac{30 \arctan\left(\frac{2c(16a^2c^2 - 8ab^2c + b^4)x + b(16a^2c^2 - 8ab^2c + b^4)}{\sqrt{1024c^5a^5 - 1280c^4b^2a^4 + 640c^3b^4a^3 - 160c^2b^6a^2 + 20cb^8a - b^{10}}}\right) b^4 c}{a^3 \sqrt{1024c^5a^5 - 1280c^4b^2a^4 + 640c^3b^4a^3 - 160c^2b^6a^2 + 20cb^8a - b^{10}}} \\
& + \frac{3 \arctan\left(\frac{2c(16a^2c^2 - 8ab^2c + b^4)x + b(16a^2c^2 - 8ab^2c + b^4)}{\sqrt{1024c^5a^5 - 1280c^4b^2a^4 + 640c^3b^4a^3 - 160c^2b^6a^2 + 20cb^8a - b^{10}}}\right) b^6}{a^4 \sqrt{1024c^5a^5 - 1280c^4b^2a^4 + 640c^3b^4a^3 - 160c^2b^6a^2 + 20cb^8a - b^{10}}}
\end{aligned}$$

Problem 130: Unable to integrate problem.

$$\int (a^2 + 2abx^{1/3} + b^2x^{2/3})^p (dx)^m dx$$

Optimal (type 5, 73 leaves, 4 steps):

$$\frac{(a^2 + 2abx^{1/3} + b^2x^{2/3})^p x (dx)^m \operatorname{hypergeom}\left(\left[-2p, 3 + 3m\right], \left[4 + 3m\right], -\frac{bx^{1/3}}{a}\right)}{(1+m) \left(1 + \frac{bx^{1/3}}{a}\right)^{2p}}$$

Result (type 8, 28 leaves):

$$\int (a^2 + 2abx^{1/3} + b^2x^{2/3})^p (dx)^m dx$$

Problem 131: Unable to integrate problem.

$$\int (a^2 + 2abx^{1/3} + b^2x^{2/3})^p x dx$$

Optimal (type 3, 275 leaves, 4 steps):

$$\begin{aligned}
& - \frac{3a^6 \left(1 + \frac{bx^{1/3}}{a}\right) (a^2 + 2abx^{1/3} + b^2x^{2/3})^p}{b^6(1+2p)} + \frac{15a^6 \left(1 + \frac{bx^{1/3}}{a}\right)^2 (a^2 + 2abx^{1/3} + b^2x^{2/3})^p}{2b^6(1+p)} \\
& - \frac{30a^6 \left(1 + \frac{bx^{1/3}}{a}\right)^3 (a^2 + 2abx^{1/3} + b^2x^{2/3})^p}{b^6(3+2p)} + \frac{15a^6 \left(1 + \frac{bx^{1/3}}{a}\right)^4 (a^2 + 2abx^{1/3} + b^2x^{2/3})^p}{b^6(2+p)} \\
& - \frac{15a^6 \left(1 + \frac{bx^{1/3}}{a}\right)^5 (a^2 + 2abx^{1/3} + b^2x^{2/3})^p}{b^6(5+2p)} + \frac{3a^6 \left(1 + \frac{bx^{1/3}}{a}\right)^6 (a^2 + 2abx^{1/3} + b^2x^{2/3})^p}{2b^6(3+p)}
\end{aligned}$$

Result(type 8, 24 leaves):

$$\int (a^2 + 2abx^{1/3} + b^2x^{2/3})^p x \, dx$$

Problem 132: Unable to integrate problem.

$$\int \frac{(a^2 + 2abx^{1/3} + b^2x^{2/3})^p}{x} \, dx$$

Optimal(type 5, 63 leaves, 3 steps):

$$\frac{3 \left(1 + \frac{bx^{1/3}}{a}\right) (a^2 + 2abx^{1/3} + b^2x^{2/3})^p \operatorname{hypergeom}\left(\left[1, 1 + 2p\right], [2 + 2p], 1 + \frac{bx^{1/3}}{a}\right)}{1 + 2p}$$

Result(type 8, 26 leaves):

$$\int \frac{(a^2 + 2abx^{1/3} + b^2x^{2/3})^p}{x} \, dx$$

Problem 137: Result is not expressed in closed-form.

$$\int \frac{x^{-1 - \frac{n}{4}}}{bx^n + cx^{2n}} \, dx$$

Optimal(type 3, 195 leaves, 14 steps):

$$\begin{aligned} & -\frac{4}{5bnx^{\frac{5n}{4}}} + \frac{4c}{b^2nx^{\frac{n}{4}}} + \frac{c^5/4 \ln\left(-\frac{b^{1/4}c^{1/4}\sqrt{2}}{x^4} + \frac{\sqrt{b}}{x^2} + \sqrt{c}\right)\sqrt{2}}{2b^9/4n} - \frac{c^5/4 \ln\left(\frac{b^{1/4}c^{1/4}\sqrt{2}}{x^4} + \frac{\sqrt{b}}{x^2} + \sqrt{c}\right)\sqrt{2}}{2b^9/4n} \\ & + \frac{c^5/4 \arctan\left(1 - \frac{b^{1/4}\sqrt{2}}{c^{1/4}x^4}\right)\sqrt{2}}{b^9/4n} - \frac{c^5/4 \arctan\left(1 + \frac{b^{1/4}\sqrt{2}}{c^{1/4}x^4}\right)\sqrt{2}}{b^9/4n} \end{aligned}$$

Result(type 7, 72 leaves):

$$\frac{4c}{b^2nx^{\frac{n}{4}}} - \frac{4}{5bn\left(\frac{n}{x^4}\right)^5} + \left(\sum_{R=\operatorname{RootOf}(b^9n^4Z^4+c^5)} -R \ln\left(x^{\frac{n}{4}} + \frac{b^7n^3R^3}{c^4}\right)\right)$$

Problem 142: Unable to integrate problem.

$$\int \frac{x^2}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}} dx$$

Optimal(type 5, 62 leaves, 2 steps):

$$\frac{x^3 (a + bx^n) \operatorname{hypergeom}\left(\left[1, \frac{3}{n}\right], \left[\frac{3+n}{n}\right], -\frac{bx^n}{a}\right)}{3a\sqrt{a^2 + 2abx^n + b^2x^{2n}}}$$

Result(type 8, 28 leaves):

$$\int \frac{x^2}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}} dx$$

Problem 143: Unable to integrate problem.

$$\int \frac{(dx)^m}{(a^2 + 2abx^n + b^2x^{2n})^{3/2}} dx$$

Optimal(type 5, 76 leaves, 2 steps):

$$\frac{(dx)^{1+m} (a + bx^n) \operatorname{hypergeom}\left(\left[3, \frac{1+m}{n}\right], \left[\frac{1+m+n}{n}\right], -\frac{bx^n}{a}\right)}{a^3 d (1+m) \sqrt{a^2 + 2abx^n + b^2x^{2n}}}$$

Result(type 8, 30 leaves):

$$\int \frac{(dx)^m}{(a^2 + 2abx^n + b^2x^{2n})^{3/2}} dx$$

Problem 145: Unable to integrate problem.

$$\int \frac{1}{x^2 (a^2 + 2abx^n + b^2x^{2n})^{3/2}} dx$$

Optimal(type 5, 62 leaves, 2 steps):

$$\frac{(a + bx^n) \operatorname{hypergeom}\left(\left[3, -\frac{1}{n}\right], \left[\frac{-1+n}{n}\right], -\frac{bx^n}{a}\right)}{a^3 x \sqrt{a^2 + 2abx^n + b^2x^{2n}}}$$

Result(type 8, 117 leaves):

$$\frac{(2bn e^{n \ln(x)} + 3an + b e^{n \ln(x)} + a) \sqrt{(b e^{n \ln(x)} + a)^2}}{2a^2 n^2 x (b e^{n \ln(x)} + a)^3} + \frac{\left(\int \frac{2n^2 + 3n + 1}{2a^2 n^2 x^2 (b e^{n \ln(x)} + a)} dx\right) \sqrt{(b e^{n \ln(x)} + a)^2}}{b e^{n \ln(x)} + a}$$

Problem 146: Unable to integrate problem.

$$\int (dx)^{-1-2n(1+p)} (a^2 + 2abx^n + b^2x^{2n})^p dx$$

Optimal(type 3, 119 leaves, 3 steps):

$$-\frac{(a+bx^n)(a^2+2abx^n+b^2x^{2n})^p}{adn(1+2p)(dx)^{2n(1+p)}} + \frac{(a^2+2abx^n+b^2x^{2n})^{1+p}}{2a^2dn(1+p)(1+2p)(dx)^{2n(1+p)}}$$

Result(type 8, 37 leaves):

$$\int (dx)^{-1-2n(1+p)} (a^2 + 2abx^n + b^2x^{2n})^p dx$$

Problem 147: Result more than twice size of optimal antiderivative.

$$\int \frac{x^{-1-n}}{a+bx^n+cx^{2n}} dx$$

Optimal(type 3, 92 leaves, 8 steps):

$$-\frac{1}{anx^n} - \frac{b \ln(x)}{a^2} + \frac{b \ln(a+bx^n+cx^{2n})}{2a^2n} - \frac{(-2ac+b^2) \operatorname{arctanh}\left(\frac{b+2cx^n}{\sqrt{-4ac+b^2}}\right)}{a^2n\sqrt{-4ac+b^2}}$$

Result(type 3, 657 leaves):

$$\begin{aligned} & -\frac{1}{anx^n} - \frac{4n^2 \ln(x) abc}{4a^3cn^2 - a^2b^2n^2} + \frac{n^2 \ln(x) b^3}{4a^3cn^2 - a^2b^2n^2} + \frac{2 \ln\left(x^n - \frac{-2abc+b^3 + \sqrt{-16a^3c^3 + 20a^2b^2c^2 - 8ab^4c + b^6}}{2c(2ac-b^2)}\right) bc}{a(4ac-b^2)n} \\ & - \frac{\ln\left(x^n - \frac{-2abc+b^3 + \sqrt{-16a^3c^3 + 20a^2b^2c^2 - 8ab^4c + b^6}}{2c(2ac-b^2)}\right) b^3}{2a^2(4ac-b^2)n} \\ & + \frac{\ln\left(x^n - \frac{-2abc+b^3 + \sqrt{-16a^3c^3 + 20a^2b^2c^2 - 8ab^4c + b^6}}{2c(2ac-b^2)}\right) \sqrt{-16a^3c^3 + 20a^2b^2c^2 - 8ab^4c + b^6}}{2a^2(4ac-b^2)n} \\ & + \frac{2 \ln\left(x^n + \frac{2abc-b^3 + \sqrt{-16a^3c^3 + 20a^2b^2c^2 - 8ab^4c + b^6}}{2c(2ac-b^2)}\right) bc}{a(4ac-b^2)n} - \frac{\ln\left(x^n + \frac{2abc-b^3 + \sqrt{-16a^3c^3 + 20a^2b^2c^2 - 8ab^4c + b^6}}{2c(2ac-b^2)}\right) b^3}{2a^2(4ac-b^2)n} \\ & - \frac{\ln\left(x^n + \frac{2abc-b^3 + \sqrt{-16a^3c^3 + 20a^2b^2c^2 - 8ab^4c + b^6}}{2c(2ac-b^2)}\right) \sqrt{-16a^3c^3 + 20a^2b^2c^2 - 8ab^4c + b^6}}{2a^2(4ac-b^2)n} \end{aligned}$$

Problem 148: Result is not expressed in closed-form.

$$\int \frac{x^{-1+\frac{n}{4}}}{a+bx^n+cx^{2n}} dx$$

Optimal (type 3, 273 leaves, 8 steps):

$$\frac{2^{2^3/4} c^{3/4} \arctan\left(\frac{2^{1/4} c^{1/4} x^{\frac{n}{4}}}{(-b-\sqrt{-4ac+b^2})^{1/4}}\right)}{n(-b-\sqrt{-4ac+b^2})^{3/4} \sqrt{-4ac+b^2}} + \frac{2^{2^3/4} c^{3/4} \operatorname{arctanh}\left(\frac{2^{1/4} c^{1/4} x^{\frac{n}{4}}}{(-b-\sqrt{-4ac+b^2})^{1/4}}\right)}{n(-b-\sqrt{-4ac+b^2})^{3/4} \sqrt{-4ac+b^2}} - \frac{2^{2^3/4} c^{3/4} \arctan\left(\frac{2^{1/4} c^{1/4} x^{\frac{n}{4}}}{(-b+\sqrt{-4ac+b^2})^{1/4}}\right)}{n\sqrt{-4ac+b^2}(-b+\sqrt{-4ac+b^2})^{3/4}} - \frac{2^{2^3/4} c^{3/4} \operatorname{arctanh}\left(\frac{2^{1/4} c^{1/4} x^{\frac{n}{4}}}{(-b+\sqrt{-4ac+b^2})^{1/4}}\right)}{n\sqrt{-4ac+b^2}(-b+\sqrt{-4ac+b^2})^{3/4}}$$

Result (type 7, 279 leaves):

$$\sum_{R=\text{RootOf}((256a^7c^4n^8-256a^6b^2c^3n^8+96a^5b^4c^2n^8-16a^4b^6cn^8+a^3b^8n^8)z^8+(-48a^3bc^3n^4+40a^2b^3c^2n^4-11ab^5cn^4+b^7n^4)z^4+c^3)} -R \ln\left(x^{\frac{n}{4}} + \left(\frac{16n^5ba^5c^2}{ac^2-b^2c}\right.\right. \\ \left.\left.- \frac{8n^5b^3a^4c}{ac^2-b^2c} + \frac{n^5b^5a^3}{ac^2-b^2c}\right) -R^5 + \left(\frac{2na^2c^2}{ac^2-b^2c} - \frac{4nab^2c}{ac^2-b^2c} + \frac{nb^4}{ac^2-b^2c}\right) -R\right)$$

Problem 149: Result is not expressed in closed-form.

$$\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}} dx$$

Optimal (type 3, 465 leaves, 14 steps):

$$\frac{2^{2/3} c^{2/3} \ln\left(2^{1/3} c^{1/3} x^{\frac{n}{3}} + (b-\sqrt{-4ac+b^2})^{1/3}\right)}{n(b-\sqrt{-4ac+b^2})^{2/3} \sqrt{-4ac+b^2}} - \frac{c^{2/3} \ln\left(2^{2/3} c^{2/3} x^{\frac{2n}{3}} - 2^{1/3} c^{1/3} x^{\frac{n}{3}} (b-\sqrt{-4ac+b^2})^{1/3} + (b-\sqrt{-4ac+b^2})^{2/3}\right)}{2n(b-\sqrt{-4ac+b^2})^{2/3} \sqrt{-4ac+b^2}}$$

$$\begin{aligned}
& - \frac{2^{2/3} c^{2/3} \arctan \left(\frac{\left(1 - \frac{2 \cdot 2^{1/3} c^{1/3} x^{n/3}}{(b - \sqrt{-4ac + b^2})^{1/3}} \right) \sqrt{3}}{3} \right) \sqrt{3}}{n (b - \sqrt{-4ac + b^2})^{2/3} \sqrt{-4ac + b^2}} - \frac{2^{2/3} c^{2/3} \ln \left(2^{1/3} c^{1/3} x^{n/3} + (b + \sqrt{-4ac + b^2})^{1/3} \right)}{n \sqrt{-4ac + b^2} (b + \sqrt{-4ac + b^2})^{2/3}} \\
& + \frac{c^{2/3} \ln \left(2^{2/3} c^{2/3} x^{2n/3} - 2^{1/3} c^{1/3} x^{n/3} (b + \sqrt{-4ac + b^2})^{1/3} + (b + \sqrt{-4ac + b^2})^{2/3} \right) 2^{2/3}}{2n \sqrt{-4ac + b^2} (b + \sqrt{-4ac + b^2})^{2/3}} \\
& + \frac{2^{2/3} c^{2/3} \arctan \left(\frac{\left(1 - \frac{2 \cdot 2^{1/3} c^{1/3} x^{n/3}}{(b + \sqrt{-4ac + b^2})^{1/3}} \right) \sqrt{3}}{3} \right) \sqrt{3}}{n \sqrt{-4ac + b^2} (b + \sqrt{-4ac + b^2})^{2/3}}
\end{aligned}$$

Result(type 7, 259 leaves):

$$\begin{aligned}
& \sum_{R=\text{RootOf}((64a^5c^3n^6 - 48a^4b^2c^2n^6 + 12a^3b^4cn^6 - a^2b^6n^6)Z^6 + (16a^2b^2cn^3 - 8ab^3cn^3 + b^5n^3)Z^3 + c^2)} -R \ln \left(x^{n/3} + \left(-\frac{16n^4ba^4c^2}{2ac^2 - b^2c} + \frac{8n^4b^3a^3c}{2ac^2 - b^2c} \right. \right. \\
& \left. \left. - \frac{n^4b^5a^2}{2ac^2 - b^2c} \right) -R^4 + \left(\frac{4na^2c^2}{2ac^2 - b^2c} - \frac{5nab^2c}{2ac^2 - b^2c} + \frac{nb^4}{2ac^2 - b^2c} \right) -R \right)
\end{aligned}$$

Problem 150: Result is not expressed in closed-form.

$$\int \frac{x^{-1 + \frac{n}{2}}}{a + bx^n + cx^{2n}} dx$$

Optimal(type 3, 129 leaves, 4 steps):

$$\frac{2 \arctan \left(\frac{x^{n/2} \sqrt{2} \sqrt{c}}{\sqrt{b - \sqrt{-4ac + b^2}}} \right) \sqrt{2} \sqrt{c}}{n \sqrt{-4ac + b^2} \sqrt{b - \sqrt{-4ac + b^2}}} - \frac{2 \arctan \left(\frac{x^{n/2} \sqrt{2} \sqrt{c}}{\sqrt{b + \sqrt{-4ac + b^2}}} \right) \sqrt{2} \sqrt{c}}{n \sqrt{-4ac + b^2} \sqrt{b + \sqrt{-4ac + b^2}}}$$

Result(type 7, 113 leaves):

$$\sum_{R=\text{RootOf}((16a^3c^2n^4 - 8a^2b^2cn^4 + ab^4n^4)Z^4 + (-4abcn^2 + b^3n^2)Z^2 + c)} -R \ln \left(x^{n/2} + \left(4a^2bn^3 - \frac{n^3b^3a}{c} \right) -R^3 + \left(2an - \frac{nb^2}{c} \right) -R \right)$$

Problem 151: Result is not expressed in closed-form.

$$\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}} dx$$

Optimal (type 3, 570 leaves, 16 steps):

$$\begin{aligned} & -\frac{3}{anx^{\frac{n}{3}}} + \frac{\ln\left(\frac{2^{1/3}a^{1/3}}{x^{\frac{n}{3}}} + (b - \sqrt{-4ac+b^2})^{1/3}\right) \left(b + \frac{2ac-b^2}{\sqrt{-4ac+b^2}}\right)^{2/3}}{2a^{4/3}n(b - \sqrt{-4ac+b^2})^{2/3}} \\ & - \frac{\ln\left(\frac{2^{2/3}a^{2/3}}{x^{\frac{2n}{3}}} - \frac{2^{1/3}a^{1/3}(b - \sqrt{-4ac+b^2})^{1/3}}{x^{\frac{n}{3}}} + (b - \sqrt{-4ac+b^2})^{2/3}\right) \left(b + \frac{2ac-b^2}{\sqrt{-4ac+b^2}}\right)^{2/3}}{4a^{4/3}n(b - \sqrt{-4ac+b^2})^{2/3}} \\ & - \frac{\arctan\left(\frac{\left(1 - \frac{2^{2/3}a^{1/3}}{x^{\frac{n}{3}}(b - \sqrt{-4ac+b^2})^{1/3}}\right)\sqrt{3}}{3}\right) \sqrt{3} \left(b + \frac{2ac-b^2}{\sqrt{-4ac+b^2}}\right)^{2/3}}{2a^{4/3}n(b - \sqrt{-4ac+b^2})^{2/3}} \\ & + \frac{\ln\left(\frac{2^{1/3}a^{1/3}}{x^{\frac{n}{3}}} + (b + \sqrt{-4ac+b^2})^{1/3}\right) \left(b + \frac{-2ac+b^2}{\sqrt{-4ac+b^2}}\right)^{2/3}}{2a^{4/3}n(b + \sqrt{-4ac+b^2})^{2/3}} \\ & - \frac{\ln\left(\frac{2^{2/3}a^{2/3}}{x^{\frac{2n}{3}}} - \frac{2^{1/3}a^{1/3}(b + \sqrt{-4ac+b^2})^{1/3}}{x^{\frac{n}{3}}} + (b + \sqrt{-4ac+b^2})^{2/3}\right) \left(b + \frac{-2ac+b^2}{\sqrt{-4ac+b^2}}\right)^{2/3}}{4a^{4/3}n(b + \sqrt{-4ac+b^2})^{2/3}} \\ & - \frac{\arctan\left(\frac{\left(1 - \frac{2^{2/3}a^{1/3}}{x^{\frac{n}{3}}(b + \sqrt{-4ac+b^2})^{1/3}}\right)\sqrt{3}}{3}\right) \sqrt{3} \left(b + \frac{-2ac+b^2}{\sqrt{-4ac+b^2}}\right)^{2/3}}{2a^{4/3}n(b + \sqrt{-4ac+b^2})^{2/3}} \end{aligned}$$

Result (type 7, 533 leaves):

$$-\frac{3}{anx^3} + \left(\sum_{R=RootOf((64a^7c^3n^6-48a^6b^2c^2n^6+12a^5b^4cn^6-a^4b^6n^6)_Z^6+(-32a^3b^3c^3n^3+32a^2b^3c^2n^3-10ab^5cn^3+b^7n^3)_Z^3+c^4)} -R \ln \left(x^{\frac{n}{3}} + \left(-\frac{64n^5a^8c^4}{2a^2c^5-4b^2ac^4+b^4c^3} + \frac{112n^5b^2a^7c^3}{2a^2c^5-4b^2ac^4+b^4c^3} - \frac{60n^5b^4a^6c^2}{2a^2c^5-4b^2ac^4+b^4c^3} + \frac{13n^5b^6a^5c}{2a^2c^5-4b^2ac^4+b^4c^3} - \frac{n^5b^8a^4}{2a^2c^5-4b^2ac^4+b^4c^3} \right) -R^5 + \left(\frac{28c^4a^4bn^2}{2a^2c^5-4b^2ac^4+b^4c^3} - \frac{63c^3a^3b^3n^2}{2a^2c^5-4b^2ac^4+b^4c^3} + \frac{42c^2a^2b^5n^2}{2a^2c^5-4b^2ac^4+b^4c^3} - \frac{11cab^7n^2}{2a^2c^5-4b^2ac^4+b^4c^3} + \frac{n^2b^9}{2a^2c^5-4b^2ac^4+b^4c^3} \right) -R^2 \right) \right)$$

Problem 152: Result is not expressed in closed-form.

$$\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n+cx^{2n}} dx$$

Optimal (type 3, 344 leaves, 10 steps):

$$-\frac{4}{anx^{\frac{n}{4}}} \frac{2^{\frac{3}{4}} \arctan \left(\frac{2^{\frac{1}{4}} a^{\frac{1}{4}}}{x^{\frac{n}{4}} (-b - \sqrt{-4ac + b^2})^{\frac{1}{4}}} \right) \left(b + \frac{-2ac + b^2}{\sqrt{-4ac + b^2}} \right)}{a^{\frac{5}{4}} n (-b - \sqrt{-4ac + b^2})^{\frac{3}{4}}} - \frac{2^{\frac{3}{4}} \operatorname{arctanh} \left(\frac{2^{\frac{1}{4}} a^{\frac{1}{4}}}{x^{\frac{n}{4}} (-b - \sqrt{-4ac + b^2})^{\frac{1}{4}}} \right) \left(b + \frac{-2ac + b^2}{\sqrt{-4ac + b^2}} \right)}{a^{\frac{5}{4}} n (-b - \sqrt{-4ac + b^2})^{\frac{3}{4}}} - \frac{2^{\frac{3}{4}} \arctan \left(\frac{2^{\frac{1}{4}} a^{\frac{1}{4}}}{x^{\frac{n}{4}} (-b + \sqrt{-4ac + b^2})^{\frac{1}{4}}} \right) \left(b + \frac{2ac - b^2}{\sqrt{-4ac + b^2}} \right)}{a^{\frac{5}{4}} n (-b + \sqrt{-4ac + b^2})^{\frac{3}{4}}} - \frac{2^{\frac{3}{4}} \operatorname{arctanh} \left(\frac{2^{\frac{1}{4}} a^{\frac{1}{4}}}{x^{\frac{n}{4}} (-b + \sqrt{-4ac + b^2})^{\frac{1}{4}}} \right) \left(b + \frac{2ac - b^2}{\sqrt{-4ac + b^2}} \right)}{a^{\frac{5}{4}} n (-b + \sqrt{-4ac + b^2})^{\frac{3}{4}}}$$

Result (type 7, 629 leaves):

$$-\frac{4}{anx^{\frac{n}{4}}} + \left(\sum_{R=RootOf((256a^9c^4n^8-256a^8b^2c^3n^8+96a^7b^4c^2n^8-16a^6b^6cn^8+a^5b^8n^8)_Z^8+(80a^4b^4c^4n^4-120a^3b^3c^3n^4+61a^2b^5c^2n^4-13ab^7cn^4+b^9n^4)_Z^4+c^5)} -R \ln \left(x^{\frac{n}{4}} + \left(-\frac{128n^7a^{10}c^5}{a^2c^6-3ab^2c^5+b^4c^4} + \frac{352n^7b^2a^9c^4}{a^2c^6-3ab^2c^5+b^4c^4} - \frac{280n^7b^4a^8c^3}{a^2c^6-3ab^2c^5+b^4c^4} + \frac{98n^7b^6a^7c^2}{a^2c^6-3ab^2c^5+b^4c^4} - \frac{16n^7b^8a^6c}{a^2c^6-3ab^2c^5+b^4c^4} + \frac{n^7b^{10}a^5}{a^2c^6-3ab^2c^5+b^4c^4} \right) -R^7 + \left(-\frac{36n^3ba^5c^5}{a^2c^6-3ab^2c^5+b^4c^4} + \frac{129n^3b^3a^4c^4}{a^2c^6-3ab^2c^5+b^4c^4} - \frac{138n^3b^5a^3c^3}{a^2c^6-3ab^2c^5+b^4c^4} + \frac{63n^3b^7a^2c^2}{a^2c^6-3ab^2c^5+b^4c^4} - \frac{13n^3b^9ac}{a^2c^6-3ab^2c^5+b^4c^4} + \frac{n^3b^{11}}{a^2c^6-3ab^2c^5+b^4c^4} \right) -R^3 \right) \right)$$

Problem 153: Unable to integrate problem.

$$\int \frac{1}{x^3 (a + bx^n + cx^{2n})} dx$$

Optimal(type 5, 130 leaves, 3 steps):

$$\frac{\text{chypergeom}\left(\left[1, -\frac{2}{n}\right], \left[\frac{-2+n}{n}\right], -\frac{2cx^n}{b - \sqrt{-4ac + b^2}}\right)}{x^2 (b^2 - 4ac - b\sqrt{-4ac + b^2})} + \frac{\text{chypergeom}\left(\left[1, -\frac{2}{n}\right], \left[\frac{-2+n}{n}\right], -\frac{2cx^n}{b + \sqrt{-4ac + b^2}}\right)}{x^2 (b^2 - 4ac + b\sqrt{-4ac + b^2})}$$

Result(type 8, 22 leaves):

$$\int \frac{1}{x^3 (a + bx^n + cx^{2n})} dx$$

Problem 156: Unable to integrate problem.

$$\int \frac{x^2}{(a + bx^n + cx^{2n})^{3/2}} dx$$

Optimal(type 6, 131 leaves, 2 steps):

$$\frac{x^3 \text{AppellF1}\left(\frac{3}{n}, \frac{3}{2}, \frac{3}{2}, \frac{3+n}{n}, -\frac{2cx^n}{b - \sqrt{-4ac + b^2}}, -\frac{2cx^n}{b + \sqrt{-4ac + b^2}}\right) \sqrt{1 + \frac{2cx^n}{b - \sqrt{-4ac + b^2}}} \sqrt{1 + \frac{2cx^n}{b + \sqrt{-4ac + b^2}}}}{3a\sqrt{a + bx^n + cx^{2n}}}$$

Result(type 8, 22 leaves):

$$\int \frac{x^2}{(a + bx^n + cx^{2n})^{3/2}} dx$$

Problem 157: Unable to integrate problem.

$$\int \frac{1}{x (a + bx^n + cx^{2n})^{3/2}} dx$$

Optimal(type 3, 88 leaves, 5 steps):

$$-\frac{\operatorname{arctanh}\left(\frac{2a + bx^n}{2\sqrt{a}\sqrt{a + bx^n + cx^{2n}}}\right)}{a^{3/2}n} + \frac{2(b^2 - 2ac + bcx^n)}{a(-4ac + b^2)n\sqrt{a + bx^n + cx^{2n}}}$$

Result(type 8, 22 leaves):

$$\int \frac{1}{x (a + bx^n + cx^{2n})^{3/2}} dx$$

Problem 158: Result more than twice size of optimal antiderivative.

$$\int (dx)^m (a + bx^n + cx^{2n}) dx$$

Optimal(type 3, 58 leaves, 6 steps):

$$\frac{bx^{1+n} (dx)^m}{1+m+n} + \frac{cx^{1+2n} (dx)^m}{1+m+2n} + \frac{a (dx)^{1+m}}{d(1+m)}$$

Result(type 3, 204 leaves):

$$\frac{1}{(1+m)(1+m+n)(1+m+2n)} \left(x \left(cm^2 (x^n)^2 + cmn (x^n)^2 + b m^2 x^n + 2 b m n x^n + 2 m c (x^n)^2 + c (x^n)^2 n + a m^2 + 3 a m n + 2 a n^2 + 2 x^n b m \right. \right. \\ \left. \left. + 2 b x^n n + c (x^n)^2 + 2 a m + 3 a n + b x^n + a \right) e^{-\frac{m(1\pi \operatorname{csgn}(1dx)^3 - 1\pi \operatorname{csgn}(1dx)^2 \operatorname{csgn}(1d) - 1\pi \operatorname{csgn}(1dx)^2 \operatorname{csgn}(1x) + 1\pi \operatorname{csgn}(1dx) \operatorname{csgn}(1d) \operatorname{csgn}(1x) - 2 \ln(x) - 2 \ln(d))}{2}} \right)$$

Problem 159: Unable to integrate problem.

$$\int \frac{(dx)^m}{(a + bx^n + cx^{2n})^3} dx$$

Optimal(type 5, 595 leaves, 6 steps):

$$\frac{(dx)^{1+m} (b^2 - 2ac + bcx^n)}{2a(-4ac + b^2)dn(a + bx^n + cx^{2n})^2} \\ - \frac{(dx)^{1+m} (4a^2c^2(1+m-4n) - 5ab^2c(1+m-3n) + b^4(1+m-2n) - bc(2ac(2+2m-7n) - b^2(1+m-2n))x^n)}{2a^2(-4ac + b^2)^2dn^2(a + bx^n + cx^{2n})} \\ - \frac{1}{2a^2(-4ac + b^2)^{5/2}d(1+m)n^2(b - \sqrt{-4ac + b^2})} \left(c(dx)^{1+m} \operatorname{hypergeom} \left(\left[1, \frac{1+m}{n} \right], \left[\frac{1+m+n}{n} \right], -\frac{2cx^n}{b - \sqrt{-4ac + b^2}} \right) \left(-b^4(1+m^2 \right. \right. \\ \left. \left. + m(2-3n) - 3n + 2n^2) + 6ab^2c(1+m^2 + m(2-4n) - 4n + 3n^2) - 8a^2c^2(1+m^2 + m(2-6n) - 6n + 8n^2) + b(2ac(2+2m-7n) \right. \right. \\ \left. \left. - b^2(1+m-2n))(1+m-n)\sqrt{-4ac + b^2} \right) \right) - \frac{1}{2a^2(-4ac + b^2)^{5/2}d(1+m)n^2(b + \sqrt{-4ac + b^2})} \left(c(dx)^{1+m} \operatorname{hypergeom} \left(\left[1, \right. \right. \right. \\ \left. \left. \frac{1+m}{n} \right], \left[\frac{1+m+n}{n} \right], -\frac{2cx^n}{b + \sqrt{-4ac + b^2}} \right) \left(b^4(1+m^2 + m(2-3n) - 3n + 2n^2) - 6ab^2c(1+m^2 + m(2-4n) - 4n + 3n^2) + 8a^2c^2(1+m^2 \right. \right. \\ \left. \left. + m(2-6n) - 6n + 8n^2) + b(2ac(2+2m-7n) - b^2(1+m-2n))(1+m-n)\sqrt{-4ac + b^2} \right) \right)$$

Result(type 8, 24 leaves):

$$\int \frac{(dx)^m}{(a + bx^n + cx^{2n})^3} dx$$

Problem 160: Unable to integrate problem.

$$\int \frac{(dx)^m}{\sqrt{a + bx^n + cx^{2n}}} dx$$

Optimal(type 6, 142 leaves, 2 steps):

$$\frac{(dx)^{1+m} \text{AppellF1}\left(\frac{1+m}{n}, \frac{1}{2}, \frac{1}{2}, \frac{1+m+n}{n}, -\frac{2cx^n}{b-\sqrt{-4ac+b^2}}, -\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right) \sqrt{1+\frac{2cx^n}{b-\sqrt{-4ac+b^2}}} \sqrt{1+\frac{2cx^n}{b+\sqrt{-4ac+b^2}}}}{d(1+m)\sqrt{a+bx^n+cx^{2n}}}$$

Result(type 8, 24 leaves):

$$\int \frac{(dx)^m}{\sqrt{a + bx^n + cx^{2n}}} dx$$

Problem 161: Unable to integrate problem.

$$\int \frac{(dx)^m}{(a + bx^n + cx^{2n})^{3/2}} dx$$

Optimal(type 6, 145 leaves, 2 steps):

$$\frac{(dx)^{1+m} \text{AppellF1}\left(\frac{1+m}{n}, \frac{3}{2}, \frac{3}{2}, \frac{1+m+n}{n}, -\frac{2cx^n}{b-\sqrt{-4ac+b^2}}, -\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right) \sqrt{1+\frac{2cx^n}{b-\sqrt{-4ac+b^2}}} \sqrt{1+\frac{2cx^n}{b+\sqrt{-4ac+b^2}}}}{ad(1+m)\sqrt{a+bx^n+cx^{2n}}}$$

Result(type 8, 24 leaves):

$$\int \frac{(dx)^m}{(a + bx^n + cx^{2n})^{3/2}} dx$$

Problem 162: Result more than twice size of optimal antiderivative.

$$\int (ex+d)^3 (a + b(ex+d)^2 + c(ex+d)^4)^2 dx$$

Optimal(type 1, 79 leaves, 4 steps):

$$\frac{a^2 (ex+d)^4}{4e} + \frac{ab (ex+d)^6}{3e} + \frac{(2ac+b^2) (ex+d)^8}{8e} + \frac{bc (ex+d)^{10}}{5e} + \frac{c^2 (ex+d)^{12}}{12e}$$

Result(type 1, 1313 leaves):

$$\frac{e^{11} c^2 x^{12}}{12} + de^{10} c^2 x^{11} + \frac{(27d^2 e^9 c^2 + e^3 (2(6cd^2 e^2 + b^2) ce^4 + 16c^2 d^2 e^6)) x^{10}}{10} + \frac{(25d^3 c^2 e^8 + 3d^2 (2(6cd^2 e^2 + b^2) ce^4 + 16c^2 d^2 e^6) + e^3 (2(4cd^3 e + 2bde) ce^4 + 8(6cd^2 e^2 + b^2) cde^3)) x^9}{9} + \frac{1}{8} ((8d^4 c^2 e^7$$

$$\begin{aligned}
& + 3d^2e(2(6cd^2e^2 + b^2e^2)ce^4 + 16c^2d^2e^6) + 3d^2e(2(4cd^3e + 2bde)ce^4 + 8(6cd^2e^2 + b^2e^2)cde^3) + e^3(2(cd^4 + bd^2 + a)ce^4 + 8(4cd^3e \\
& + 2bde)cde^3 + (6cd^2e^2 + b^2e^2)^2)x^8) + \frac{1}{7}((d^3(2(6cd^2e^2 + b^2e^2)ce^4 + 16c^2d^2e^6) + 3d^2e(2(4cd^3e + 2bde)ce^4 + 8(6cd^2e^2 \\
& + b^2e^2)cde^3) + 3d^2e(2(cd^4 + bd^2 + a)ce^4 + 8(4cd^3e + 2bde)cde^3 + (6cd^2e^2 + b^2e^2)^2) + e^3(8(cd^4 + bd^2 + a)cde^3 + 2(4cd^3e \\
& + 2bde)(6cd^2e^2 + b^2e^2)))x^7) + \frac{1}{6}((d^3(2(4cd^3e + 2bde)ce^4 + 8(6cd^2e^2 + b^2e^2)cde^3) + 3d^2e(2(cd^4 + bd^2 + a)ce^4 + 8(4cd^3e \\
& + 2bde)cde^3 + (6cd^2e^2 + b^2e^2)^2) + 3d^2e(8(cd^4 + bd^2 + a)cde^3 + 2(4cd^3e + 2bde)(6cd^2e^2 + b^2e^2)) + e^3(2(cd^4 + bd^2 + a)(6cd^2e^2 \\
& + b^2e^2) + (4cd^3e + 2bde)^2))x^6) + \frac{1}{5}((d^3(2(cd^4 + bd^2 + a)ce^4 + 8(4cd^3e + 2bde)cde^3 + (6cd^2e^2 + b^2e^2)^2) + 3d^2e(8(cd^4 + bd^2 \\
& + a)cde^3 + 2(4cd^3e + 2bde)(6cd^2e^2 + b^2e^2)) + 3d^2e(2(cd^4 + bd^2 + a)(6cd^2e^2 + b^2e^2) + (4cd^3e + 2bde)^2) + 2e^3(cd^4 + bd^2 \\
& + a)(4cd^3e + 2bde))x^5) + \frac{1}{4}((d^3(8(cd^4 + bd^2 + a)cde^3 + 2(4cd^3e + 2bde)(6cd^2e^2 + b^2e^2)) + 3d^2e(2(cd^4 + bd^2 + a)(6cd^2e^2 \\
& + b^2e^2) + (4cd^3e + 2bde)^2) + 6d^2e(cd^4 + bd^2 + a)(4cd^3e + 2bde) + e^3(cd^4 + bd^2 + a)^2)x^4) \\
& + \frac{(d^3(2(cd^4 + bd^2 + a)(6cd^2e^2 + b^2e^2) + (4cd^3e + 2bde)^2) + 6d^2e(cd^4 + bd^2 + a)(4cd^3e + 2bde) + 3d^2e(cd^4 + bd^2 + a)^2)x^3}{3} \\
& + \frac{(2d^3(cd^4 + bd^2 + a)(4cd^3e + 2bde) + 3d^2e(cd^4 + bd^2 + a)^2)x^2}{2} + d^3(cd^4 + bd^2 + a)^2x
\end{aligned}$$

Problem 163: Result more than twice size of optimal antiderivative.

$$\int (ex + d)^3 (a + b(ex + d)^2 + c(ex + d)^4)^3 dx$$

Optimal (type 1, 124 leaves, 4 steps):

$$\begin{aligned}
& \frac{a^3 (ex + d)^4}{4e} + \frac{a^2 b (ex + d)^6}{2e} + \frac{3a(ac + b^2)(ex + d)^8}{8e} + \frac{b(6ac + b^2)(ex + d)^{10}}{10e} + \frac{c(ac + b^2)(ex + d)^{12}}{4e} + \frac{3bc^2(ex + d)^{14}}{14e} \\
& + \frac{c^3(ex + d)^{16}}{16e}
\end{aligned}$$

Result (type ?, 7549 leaves): Display of huge result suppressed!

Problem 164: Result is not expressed in closed-form.

$$\int \frac{(ex + d)^3}{a + b(ex + d)^2 + c(ex + d)^4} dx$$

Optimal (type 3, 73 leaves, 6 steps):

$$\frac{\ln(a + b(ex + d)^2 + c(ex + d)^4)}{4ce} + \frac{b \operatorname{arctanh}\left(\frac{b + 2c(ex + d)^2}{\sqrt{-4ac + b^2}}\right)}{2ce\sqrt{-4ac + b^2}}$$

Result (type 7, 150 leaves):

$$\sum_{R=\text{RootOf}(c^4 Z^4+4cd^3 Z^3+(6cd^2e^2+be^2) Z^2+(4cd^3e+2bde) Z+cd^4+bd^2+a)} \frac{(-R^3e^3+3R^2de^2+3Rd^2e+d^3)\ln(x-R)}{2ce^3R^3+6cde^2R^2+6cd^2eR+2cd^3+beR+bd} \frac{1}{2e}$$

Problem 165: Result is not expressed in closed-form.

$$\int \frac{1}{(ex+d)(a+b(ex+d)^2+c(ex+d)^4)} dx$$

Optimal(type 3, 86 leaves, 8 steps):

$$\frac{\ln(ex+d)}{ae} - \frac{\ln(a+b(ex+d)^2+c(ex+d)^4)}{4ae} + \frac{b \operatorname{arctanh}\left(\frac{b+2c(ex+d)^2}{\sqrt{-4ac+b^2}}\right)}{2ae\sqrt{-4ac+b^2}}$$

Result(type 7, 183 leaves):

$$\sum_{R=\text{RootOf}(c^4 Z^4+4cd^3 Z^3+(6cd^2e^2+be^2) Z^2+(4cd^3e+2bde) Z+cd^4+bd^2+a)} \frac{(-ce^3R^3-3cde^2R^2+e(-3cd^2-b)R-cd^3-bd)\ln(x-R)}{2ce^3R^3+6cde^2R^2+6cd^2eR+2cd^3+beR+bd} \frac{1}{2ae} + \frac{\ln(ex+d)}{ae}$$

Problem 166: Result is not expressed in closed-form.

$$\int \frac{1}{(ex+d)^2(a+b(ex+d)^2+c(ex+d)^4)} dx$$

Optimal(type 3, 159 leaves, 5 steps):

$$-\frac{1}{ae(ex+d)} - \frac{\operatorname{arctan}\left(\frac{(ex+d)\sqrt{2}\sqrt{c}}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)\sqrt{c}\left(1+\frac{b}{\sqrt{-4ac+b^2}}\right)\sqrt{2}}{2ae\sqrt{b-\sqrt{-4ac+b^2}}} - \frac{\operatorname{arctan}\left(\frac{(ex+d)\sqrt{2}\sqrt{c}}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)\sqrt{c}\left(1-\frac{b}{\sqrt{-4ac+b^2}}\right)\sqrt{2}}{2ae\sqrt{b+\sqrt{-4ac+b^2}}}$$

Result(type 7, 167 leaves):

$$\sum_{R=\text{RootOf}(c^4 Z^4+4cd^3 Z^3+(6cd^2e^2+be^2) Z^2+(4cd^3e+2bde) Z+cd^4+bd^2+a)} \frac{(-R^2ce^2-2Rcde-cd^2-b)\ln(x-R)}{2ce^3R^3+6cde^2R^2+6cd^2eR+2cd^3+beR+bd} \frac{1}{2ae} - \frac{1}{ae(ex+d)}$$

Problem 167: Result is not expressed in closed-form.

$$\int \frac{(ex+d)^4}{(a+b(ex+d)^2+c(ex+d)^4)^2} dx$$

Optimal(type 3, 227 leaves, 5 steps):

$$\frac{(ex+d)(2a+b(ex+d)^2)}{2(-4ac+b^2)e(a+b(ex+d)^2+c(ex+d)^4)} + \frac{\arctan\left(\frac{(ex+d)\sqrt{2}\sqrt{c}}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)\left(b+\frac{-4ac-b^2}{\sqrt{-4ac+b^2}}\right)\sqrt{2}}{4(-4ac+b^2)e\sqrt{c}\sqrt{b-\sqrt{-4ac+b^2}}}$$

$$+ \frac{\arctan\left(\frac{(ex+d)\sqrt{2}\sqrt{c}}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)\left(b^2+4ac+b\sqrt{-4ac+b^2}\right)\sqrt{2}}{4(-4ac+b^2)^{3/2}e\sqrt{c}\sqrt{b+\sqrt{-4ac+b^2}}}$$

Result(type 7, 322 leaves):

$$\frac{-\frac{be^2x^3}{2(4ac-b^2)} - \frac{3dbex^2}{2(4ac-b^2)} - \frac{(3bd^2+2a)x}{2(4ac-b^2)} - \frac{d(bd^2+2a)}{2e(4ac-b^2)}}{ce^4x^4+4cde^3x^3+6cd^2e^2x^2+4cd^3ex+be^2x^2+cd^4+2bdex+bd^2+a}$$

$$+ \frac{1}{4e} \left(\sum_{R=\text{RootOf}(ce^4Z^4+4cde^3Z^3+(6cd^2e^2+be^2)Z^2+(4cd^3e+2bde)Z+cd^4+bd^2+a)} \frac{(-R^2be^2-2Rbde-bd^2+2a)\ln(x-R)}{(4ac-b^2)(2ce^3R^3+6cde^2R^2+6cd^2eR+2cd^3+beR+bd)} \right)$$

Problem 168: Result is not expressed in closed-form.

$$\int \frac{(ex+d)^3}{(a+b(ex+d)^2+c(ex+d)^4)^2} dx$$

Optimal(type 3, 91 leaves, 5 steps):

$$\frac{2a+b(ex+d)^2}{2(-4ac+b^2)e(a+b(ex+d)^2+c(ex+d)^4)} - \frac{b \operatorname{arctanh}\left(\frac{b+2c(ex+d)^2}{\sqrt{-4ac+b^2}}\right)}{(-4ac+b^2)^{3/2}e}$$

Result(type 7, 275 leaves):

$$\frac{-\frac{bex^2}{2(4ac-b^2)} - \frac{bdx}{4ac-b^2} - \frac{bd^2+2a}{2e(4ac-b^2)}}{ce^4x^4+4cde^3x^3+6cd^2e^2x^2+4cd^3ex+be^2x^2+cd^4+2bdex+bd^2+a} + \frac{1}{2e} \left(b \left(\sum_{R=\text{RootOf}(ce^4Z^4+4cde^3Z^3+(6cd^2e^2+be^2)Z^2+(4cd^3e+2bde)Z+cd^4+bd^2+a)} \right. \right.$$

$$\left. \left. \left. \frac{(-e_R - d) \ln(x - R)}{(4ac - b^2) (2ce^3 R^3 + 6cde^2 R^2 + 6cd^2 e_R + 2cd^3 + be_R + bd)} \right) \right) \right)$$

Problem 169: Result is not expressed in closed-form.

$$\int \frac{(ex + d)^2}{(a + b(ex + d)^2 + c(ex + d)^4)^2} dx$$

Optimal (type 3, 213 leaves, 5 steps):

$$\begin{aligned} & - \frac{(ex + d) (b + 2c(ex + d)^2)}{2(-4ac + b^2) e (a + b(ex + d)^2 + c(ex + d)^4)} + \frac{\arctan\left(\frac{(ex + d) \sqrt{2} \sqrt{c}}{\sqrt{b - \sqrt{-4ac + b^2}}}\right) \sqrt{c} (2b - \sqrt{-4ac + b^2}) \sqrt{2}}{2(-4ac + b^2)^{3/2} e \sqrt{b - \sqrt{-4ac + b^2}}} \\ & - \frac{\arctan\left(\frac{(ex + d) \sqrt{2} \sqrt{c}}{\sqrt{b + \sqrt{-4ac + b^2}}}\right) \sqrt{c} (2b + \sqrt{-4ac + b^2}) \sqrt{2}}{2(-4ac + b^2)^{3/2} e \sqrt{b + \sqrt{-4ac + b^2}}} \end{aligned}$$

Result (type 7, 318 leaves):

$$\begin{aligned} & \frac{ce^2 x^3}{4ac - b^2} + \frac{3dce x^2}{4ac - b^2} + \frac{(6cd^2 + b)x}{2(4ac - b^2)} + \frac{d(2cd^2 + b)}{2e(4ac - b^2)} \\ & \frac{ce^4 x^4 + 4cde^3 x^3 + 6cd^2 e^2 x^2 + 4cd^3 ex + be^2 x^2 + cd^4 + 2bdex + bd^2 + a}{4ac - b^2} \\ & + \frac{1}{4e} \left(\sum_{R=\text{RootOf}(ce^4 Z^4 + 4cde^3 Z^3 + (6cd^2 e^2 + be^2) Z^2 + (4cd^3 e + 2bde) Z + cd^4 + bd^2 + a)} \frac{(2R^2 ce^2 + 4Rcde + 2cd^2 - b) \ln(x - R)}{(4ac - b^2) (2ce^3 R^3 + 6cde^2 R^2 + 6cd^2 e_R + 2cd^3 + be_R + bd)} \right) \end{aligned}$$

Problem 170: Result is not expressed in closed-form.

$$\int \frac{ex + d}{(a + b(ex + d)^2 + c(ex + d)^4)^2} dx$$

Optimal (type 3, 92 leaves, 5 steps):

$$\frac{-b - 2c(ex + d)^2}{2(-4ac + b^2) e (a + b(ex + d)^2 + c(ex + d)^4)} + \frac{2c \operatorname{arctanh}\left(\frac{b + 2c(ex + d)^2}{\sqrt{-4ac + b^2}}\right)}{(-4ac + b^2)^{3/2} e}$$

Result(type 7, 269 leaves):

$$\frac{\frac{cex^2}{4ac-b^2} + \frac{2cdx}{4ac-b^2} + \frac{2cd^2+b}{2e(4ac-b^2)}}{ce^4x^4 + 4cde^3x^3 + 6cd^2e^2x^2 + 4cd^3ex + be^2x^2 + cd^4 + 2bdex + bd^2 + a} + \frac{1}{e} \left(c \left(\sum_{R=\text{RootOf}(ce^4Z^4 + 4cde^3Z^3 + (6cd^2e^2 + be^2)Z^2 + (4cd^3e + 2bde)Z + cd^4 + bd^2 + a)} \frac{(eR+d) \ln(x-R)}{(4ac-b^2)(2ce^3R^3 + 6cde^2R^2 + 6cd^2eR + 2cd^3 + beR + bd)} \right) \right)$$

Problem 171: Result is not expressed in closed-form.

$$\int \frac{(ex+d)^2}{(a+b(ex+d)^2+c(ex+d)^4)^3} dx$$

Optimal(type 3, 319 leaves, 6 steps):

$$\begin{aligned} & -\frac{(ex+d)(b+2c(ex+d)^2)}{4(-4ac+b^2)e(a+b(ex+d)^2+c(ex+d)^4)^2} + \frac{(ex+d)(b(8ac+b^2)+c(20ac+b^2)(ex+d)^2)}{8a(-4ac+b^2)^2e(a+b(ex+d)^2+c(ex+d)^4)} \\ & + \frac{\arctan\left(\frac{(ex+d)\sqrt{2}\sqrt{c}}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)\sqrt{c}\left(b^2+20ac+\frac{b(-52ac+b^2)}{\sqrt{-4ac+b^2}}\right)\sqrt{2}}{16a(-4ac+b^2)^2e\sqrt{b-\sqrt{-4ac+b^2}}} \\ & + \frac{\arctan\left(\frac{(ex+d)\sqrt{2}\sqrt{c}}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)\sqrt{c}\left(b^2+20ac-\frac{b(-52ac+b^2)}{\sqrt{-4ac+b^2}}\right)\sqrt{2}}{16a(-4ac+b^2)^2e\sqrt{b+\sqrt{-4ac+b^2}}} \end{aligned}$$

Result(type 7, 884 leaves):

$$\begin{aligned} & \left(\frac{c^2e^6(20ac+b^2)x^7}{8(16a^2c^2-8ab^2c+b^4)a} + \frac{7c^2de^5(20ac+b^2)x^6}{8(16a^2c^2-8ab^2c+b^4)a} + \frac{(420a^2c^2d^2+21b^2cd^2+28abc+2b^3)ce^4x^5}{8(16a^2c^2-8ab^2c+b^4)a} \right. \\ & + \frac{5cde^3(140a^2c^2d^2+7b^2cd^2+28abc+2b^3)x^4}{8(16a^2c^2-8ab^2c+b^4)a} + \frac{e^2(700a^3c^3d^4+35b^2c^2d^4+280abc^2d^2+20b^3cd^2+36a^2c^2+5ab^2c+b^4)x^3}{8(16a^2c^2-8ab^2c+b^4)a} \\ & + \frac{de(420a^3c^3d^4+21b^2c^2d^4+280abc^2d^2+20b^3cd^2+108a^2c^2+15ab^2c+3b^4)x^2}{8(16a^2c^2-8ab^2c+b^4)a} \\ & + \frac{(140a^3c^3d^6+7b^2c^2d^6+140abc^2d^4+10b^3cd^4+108a^2c^2d^2+15ab^2cd^2+3b^4d^2+16a^2bc-ab^3)x}{8(16a^2c^2-8ab^2c+b^4)a} \\ & \left. + \frac{d(20a^3c^3d^6+b^2c^2d^6+28abc^2d^4+2b^3cd^4+36a^2c^2d^2+5ab^2cd^2+b^4d^2+16a^2bc-ab^3)}{8e(16a^2c^2-8ab^2c+b^4)a} \right) / (ce^4x^4 + 4cde^3x^3 + 6cd^2e^2x^2) \end{aligned}$$

$$+4cd^3ex + be^2x^2 + cd^4 + 2bdex + bd^2 + a)^2 + \frac{1}{16ae} \left(\sum_{R=\text{RootOf}(ce^4Z^4 + 4cde^3Z^3 + (6cd^2e^2 + be^2)Z^2 + (4cd^3e + 2bde)Z + cd^4 + bd^2 + a)} \frac{(ce^2(20ac + b^2)R^2 + 2cde(20ac + b^2)R + 20ac^2d^2 + b^2cd^2 - 16abc + b^3) \ln(x - R)}{(16a^2c^2 - 8ab^2c + b^4)(2ce^3R^3 + 6cd^2eR^2 + 6cd^2eR + 2cd^3 + beR + bd)} \right)$$

Problem 172: Result is not expressed in closed-form.

$$\int \frac{(efx + df)^3}{a + b(ex + d)^2 + c(ex + d)^4} dx$$

Optimal(type 3, 79 leaves, 6 steps):

$$\frac{f^3 \ln(a + b(ex + d)^2 + c(ex + d)^4)}{4ce} + \frac{bf^3 \operatorname{arctanh}\left(\frac{b + 2c(ex + d)^2}{\sqrt{-4ac + b^2}}\right)}{2ce\sqrt{-4ac + b^2}}$$

Result(type 7, 153 leaves):

$$f^3 \left(\frac{\sum_{R=\text{RootOf}(ce^4Z^4 + 4cde^3Z^3 + (6cd^2e^2 + be^2)Z^2 + (4cd^3e + 2bde)Z + cd^4 + bd^2 + a)} \frac{(-R^3e^3 + 3R^2de^2 + 3Rd^2e + d^3) \ln(x - R)}{2ce^3R^3 + 6cd^2eR^2 + 6cd^2eR + 2cd^3 + beR + bd}}{2e} \right)$$

Problem 173: Result is not expressed in closed-form.

$$\int \frac{1}{(efx + df)^2 (a + b(ex + d)^2 + c(ex + d)^4)} dx$$

Optimal(type 3, 168 leaves, 5 steps):

$$-\frac{1}{ae^2(ex + d)} - \frac{\operatorname{arctan}\left(\frac{(ex + d)\sqrt{2}\sqrt{c}}{\sqrt{b - \sqrt{-4ac + b^2}}}\right) \sqrt{c} \left(1 + \frac{b}{\sqrt{-4ac + b^2}}\right) \sqrt{2}}{2ae^2\sqrt{b - \sqrt{-4ac + b^2}}} - \frac{\operatorname{arctan}\left(\frac{(ex + d)\sqrt{2}\sqrt{c}}{\sqrt{b + \sqrt{-4ac + b^2}}}\right) \sqrt{c} \left(1 - \frac{b}{\sqrt{-4ac + b^2}}\right) \sqrt{2}}{2ae^2\sqrt{b + \sqrt{-4ac + b^2}}}$$

Result(type 7, 173 leaves):

$$\frac{\sum_{R=\text{RootOf}(ce^4Z^4 + 4cde^3Z^3 + (6cd^2e^2 + be^2)Z^2 + (4cd^3e + 2bde)Z + cd^4 + bd^2 + a)} \frac{(-R^2ce^2 - 2Rcde - cd^2 - b) \ln(x - R)}{2f^2ae}}{2f^2ae} - \frac{1}{ae^2(ex + d)}$$

Problem 174: Result is not expressed in closed-form.

$$\int \frac{1}{(efx+df)^2 (a+b(ex+d)^2+c(ex+d)^4)^2} dx$$

Optimal (type 3, 312 leaves, 6 steps):

$$\begin{aligned} & \frac{10ac-3b^2}{2a^2(-4ac+b^2)ef^2(ex+d)} + \frac{b^2-2ac+bc(ex+d)^2}{2a(-4ac+b^2)ef^2(ex+d)(a+b(ex+d)^2+c(ex+d)^4)} \\ & - \frac{\arctan\left(\frac{(ex+d)\sqrt{2}\sqrt{c}}{\sqrt{b-\sqrt{-4ac+b^2}}}\right) \sqrt{c} (3b^3-16abc+(-10ac+3b^2)\sqrt{-4ac+b^2}) \sqrt{2}}{4a^2(-4ac+b^2)^{3/2}ef^2\sqrt{b-\sqrt{-4ac+b^2}}} \\ & + \frac{\arctan\left(\frac{(ex+d)\sqrt{2}\sqrt{c}}{\sqrt{b+\sqrt{-4ac+b^2}}}\right) \sqrt{c} (3b^3-16abc-(-10ac+3b^2)\sqrt{-4ac+b^2}) \sqrt{2}}{4a^2(-4ac+b^2)^{3/2}ef^2\sqrt{b+\sqrt{-4ac+b^2}}} \end{aligned}$$

Result (type 7, 1345 leaves):

$$\begin{aligned} & - \frac{c^2 e^2 x^3}{f^2 a (ce^4 x^4 + 4cde^3 x^3 + 6cd^2 e^2 x^2 + 4cd^3 ex + b^2 e^2 x^2 + cd^4 + 2bdex + bd^2 + a) (4ac - b^2)} \\ & + \frac{c^2 e^2 x^3 b^2}{2f^2 a^2 (ce^4 x^4 + 4cde^3 x^3 + 6cd^2 e^2 x^2 + 4cd^3 ex + b^2 e^2 x^2 + cd^4 + 2bdex + bd^2 + a) (4ac - b^2)} \\ & - \frac{3dc^2 ex^2}{f^2 a (ce^4 x^4 + 4cde^3 x^3 + 6cd^2 e^2 x^2 + 4cd^3 ex + b^2 e^2 x^2 + cd^4 + 2bdex + bd^2 + a) (4ac - b^2)} \\ & + \frac{3dce^2 x^2 b^2}{2f^2 a^2 (ce^4 x^4 + 4cde^3 x^3 + 6cd^2 e^2 x^2 + 4cd^3 ex + b^2 e^2 x^2 + cd^4 + 2bdex + bd^2 + a) (4ac - b^2)} \\ & - \frac{3xc^2 d^2}{f^2 a (ce^4 x^4 + 4cde^3 x^3 + 6cd^2 e^2 x^2 + 4cd^3 ex + b^2 e^2 x^2 + cd^4 + 2bdex + bd^2 + a) (4ac - b^2)} \\ & + \frac{3xb^2 cd^2}{2f^2 a^2 (ce^4 x^4 + 4cde^3 x^3 + 6cd^2 e^2 x^2 + 4cd^3 ex + b^2 e^2 x^2 + cd^4 + 2bdex + bd^2 + a) (4ac - b^2)} \\ & - \frac{3xbc}{2f^2 a (ce^4 x^4 + 4cde^3 x^3 + 6cd^2 e^2 x^2 + 4cd^3 ex + b^2 e^2 x^2 + cd^4 + 2bdex + bd^2 + a) (4ac - b^2)} \\ & + \frac{xb^3}{2f^2 a^2 (ce^4 x^4 + 4cde^3 x^3 + 6cd^2 e^2 x^2 + 4cd^3 ex + b^2 e^2 x^2 + cd^4 + 2bdex + bd^2 + a) (4ac - b^2)} \\ & - \frac{d^3 c^2}{f^2 a (ce^4 x^4 + 4cde^3 x^3 + 6cd^2 e^2 x^2 + 4cd^3 ex + b^2 e^2 x^2 + cd^4 + 2bdex + bd^2 + a) e (4ac - b^2)} \\ & + \frac{d^3 b^2 c}{2f^2 a^2 (ce^4 x^4 + 4cde^3 x^3 + 6cd^2 e^2 x^2 + 4cd^3 ex + b^2 e^2 x^2 + cd^4 + 2bdex + bd^2 + a) e (4ac - b^2)} \end{aligned}$$

$$\begin{aligned}
& - \frac{3dbc}{2f^2 a (ce^4 x^4 + 4cde^3 x^3 + 6cd^2 e^2 x^2 + 4cd^3 ex + be^2 x^2 + cd^4 + 2bdex + bd^2 + a) e (4ac - b^2)} \\
& + \frac{db^3}{2f^2 a^2 (ce^4 x^4 + 4cde^3 x^3 + 6cd^2 e^2 x^2 + 4cd^3 ex + be^2 x^2 + cd^4 + 2bdex + bd^2 + a) e (4ac - b^2)} - \frac{1}{4f^2 a^2 e} \left(\right. \\
& \left. \sum_{R=\text{RootOf}(ce^4 Z^4 + 4cde^3 Z^3 + (6cd^2 e^2 + be^2) Z^2 + (4cd^3 e + 2bde) Z + cd^4 + bd^2 + a)} \frac{(ce^2 (10ac - 3b^2) R^2 + 2cde (10ac - 3b^2) R + 10a^2 d^2 - 3b^2 cd^2 + 13abc - 3b^3) \ln(x - R)}{(4ac - b^2) (2ce^3 R^3 + 6cd^2 e^2 R^2 + 6cd^2 e R + 2cd^3 + be R + bd)} \right) - \frac{1}{f^2 a^2 e (ex + d)}
\end{aligned}$$

Problem 175: Result is not expressed in closed-form.

$$\int \frac{(efx + df)^2}{(a + b(ex + d)^2 + c(ex + d)^4)^3} dx$$

Optimal (type 3, 331 leaves, 6 steps):

$$\begin{aligned}
& - \frac{f^2 (ex + d) (b + 2c(ex + d)^2)}{4(-4ac + b^2) e (a + b(ex + d)^2 + c(ex + d)^4)^2} + \frac{f^2 (ex + d) (b(8ac + b^2) + c(20ac + b^2)(ex + d)^2)}{8a(-4ac + b^2)^2 e (a + b(ex + d)^2 + c(ex + d)^4)} \\
& + \frac{f^2 \arctan\left(\frac{(ex + d) \sqrt{2} \sqrt{c}}{\sqrt{b - \sqrt{-4ac + b^2}}}\right) \sqrt{c} \left(b^2 + 20ac + \frac{b(-52ac + b^2)}{\sqrt{-4ac + b^2}}\right) \sqrt{2}}{16a(-4ac + b^2)^2 e \sqrt{b - \sqrt{-4ac + b^2}}} \\
& + \frac{f^2 \arctan\left(\frac{(ex + d) \sqrt{2} \sqrt{c}}{\sqrt{b + \sqrt{-4ac + b^2}}}\right) \sqrt{c} \left(b^2 + 20ac - \frac{b(-52ac + b^2)}{\sqrt{-4ac + b^2}}\right) \sqrt{2}}{16a(-4ac + b^2)^2 e \sqrt{b + \sqrt{-4ac + b^2}}}
\end{aligned}$$

Result (type ?, 4750 leaves): Display of huge result suppressed!

Problem 176: Result is not expressed in closed-form.

$$\int \frac{efx + df}{(a + b(ex + d)^2 + c(ex + d)^4)^3} dx$$

Optimal (type 3, 145 leaves, 6 steps):

$$- \frac{f(b + 2c(ex + d)^2)}{4(-4ac + b^2) e (a + b(ex + d)^2 + c(ex + d)^4)^2} + \frac{3cf(b + 2c(ex + d)^2)}{2(-4ac + b^2)^2 e (a + b(ex + d)^2 + c(ex + d)^4)} - \frac{6c^2 f \operatorname{arctanh}\left(\frac{b + 2c(ex + d)^2}{\sqrt{-4ac + b^2}}\right)}{(-4ac + b^2)^{5/2} e}$$

Result (type ?, 2131 leaves): Display of huge result suppressed!

Problem 177: Unable to integrate problem.

$$\int \frac{x^2}{\sqrt{a + b(ex+d)^3 + c(ex+d)^6}} dx$$

Optimal (type 6, 344 leaves, 10 steps):

$$\frac{\operatorname{arctanh}\left(\frac{b + 2c(ex+d)^3}{2\sqrt{c}\sqrt{a + b(ex+d)^3 + c(ex+d)^6}}\right)}{3e^3\sqrt{c}} + \frac{d^2(ex+d) \operatorname{AppellF1}\left(\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, -\frac{2c(ex+d)^3}{b - \sqrt{-4ac + b^2}}, -\frac{2c(ex+d)^3}{b + \sqrt{-4ac + b^2}}\right) \sqrt{1 + \frac{2c(ex+d)^3}{b - \sqrt{-4ac + b^2}}} \sqrt{1 + \frac{2c(ex+d)^3}{b + \sqrt{-4ac + b^2}}}}{e^3\sqrt{a + b(ex+d)^3 + c(ex+d)^6}} - \frac{d(ex+d)^2 \operatorname{AppellF1}\left(\frac{2}{3}, \frac{1}{2}, \frac{1}{2}, \frac{5}{3}, -\frac{2c(ex+d)^3}{b - \sqrt{-4ac + b^2}}, -\frac{2c(ex+d)^3}{b + \sqrt{-4ac + b^2}}\right) \sqrt{1 + \frac{2c(ex+d)^3}{b - \sqrt{-4ac + b^2}}} \sqrt{1 + \frac{2c(ex+d)^3}{b + \sqrt{-4ac + b^2}}}}{e^3\sqrt{a + b(ex+d)^3 + c(ex+d)^6}}$$

Result (type 8, 28 leaves):

$$\int \frac{x^2}{\sqrt{a + b(ex+d)^3 + c(ex+d)^6}} dx$$

Problem 178: Result more than twice size of optimal antiderivative.

$$\int (2+3x)^6 (1 + (2+3x)^7 + (2+3x)^{14}) dx$$

Optimal (type 1, 28 leaves, 3 steps):

$$\frac{(2+3x)^7}{21} + \frac{(2+3x)^{14}}{42} + \frac{(2+3x)^{21}}{63}$$

Result (type 1, 104 leaves):

$$\frac{1162261467}{7}x^{21} + 2324522934x^{20} + 15496819560x^{19} + 65431015920x^{18} + 196293047760x^{17} + 444930908256x^{16} + 790988281344x^{15} + \frac{15819767221203}{14}x^{14} + 1318314865122x^{13} + 1269491970942x^{12} + 1015602174288x^{11} + 677082445416x^{10} + 376174427616x^9 + 173635132896x^8 + 66158154783x^7 + 20588764518x^6 + 5149786572x^5 + 1010576952x^4 + 149902032x^3 + 15808800x^2 + 1056832x$$

Problem 179: Result more than twice size of optimal antiderivative.

$$\int (2+3x)^6 (1 + (2+3x)^7 + (2+3x)^{14})^2 dx$$

Optimal (type 1, 46 leaves, 4 steps):

$$\frac{(2+3x)^7}{21} + \frac{(2+3x)^{14}}{21} + \frac{(2+3x)^{21}}{21} + \frac{(2+3x)^{28}}{42} + \frac{(2+3x)^{35}}{105}$$

Result(type 1, 174 leaves):

$$\begin{aligned}
 & 17451466816x + 7299544818384x^3 + 6077684727888102x^6 + 197897276851452864x^8 + \frac{4057390785756924}{5}x^5 + 443569828128x^2 + 87406679578680x^4 \\
 & + 37727143432895007x^7 + 889942562270387136x^9 + 872775774067455498528x^{16} + \frac{17344958593049772048}{5}x^{10} + 465517091041681015296x^{15} \\
 & + 221699757548270194389x^{14} + 94069263918929616324x^{13} + 3534290697929473864098x^{20} + 2945285062308448290360x^{19} \\
 & + 2194577166014752240080x^{18} + 1463104032160519033200x^{17} + \frac{26506949038858918036881}{7}x^{21} + 11821487501620716192x^{11} \\
 & + 35454069480572048124x^{12} + 11118121133111046x^{34} + 126005372841925188x^{33} + 924039400840784712x^{32} + 4928210137817518464x^{31} \\
 & + \frac{101849676181562048256}{5}x^{30} + 67899784121041365504x^{29} + \frac{2625458326972530284475}{14}x^{28} + 437576396725285446564x^{27} \\
 & + 875152864622814086340x^{26} + \frac{7584660010542711771792}{5}x^{25} + 2298383223254096766840x^{24} + 3064515076512846852480x^{23} \\
 & + 3614565944605222108800x^{22} + \frac{16677181699666569}{35}x^{35}
 \end{aligned}$$

Test results for the 27 problems in "1.2.3.3 (d+e x^n)^q (a+b x^n+c x^(2 n))^p.txt"

Problem 2: Result is not expressed in closed-form.

$$\int \frac{ex^4 + d}{cx^8 + a} dx$$

Optimal(type 3, 518 leaves, 19 steps):

$$\begin{aligned}
 & \frac{\arctan\left(\frac{-2c^{1/8}x + a^{1/8}\sqrt{2-\sqrt{2}}}{a^{1/8}\sqrt{2+\sqrt{2}}}\right) (-e\sqrt{a} + d(1+\sqrt{2})\sqrt{c})\sqrt{2-\sqrt{2}}}{8a^{7/8}c^{5/8}} \\
 & + \frac{\arctan\left(\frac{2c^{1/8}x + a^{1/8}\sqrt{2-\sqrt{2}}}{a^{1/8}\sqrt{2+\sqrt{2}}}\right) (-e\sqrt{a} + d(1+\sqrt{2})\sqrt{c})\sqrt{2-\sqrt{2}}}{8a^{7/8}c^{5/8}} \\
 & + \frac{\ln\left(a^{1/4} + c^{1/4}x^2 - c^{1/8}a^{1/8}\sqrt{2-\sqrt{2}}x\right) (-e\sqrt{a} + d(1-\sqrt{2})\sqrt{c})}{8a^{7/8}c^{5/8}\sqrt{4-2\sqrt{2}}} \\
 & - \frac{\ln\left(a^{1/4} + c^{1/4}x^2 + c^{1/8}a^{1/8}\sqrt{2-\sqrt{2}}x\right) (-e\sqrt{a} + d(1-\sqrt{2})\sqrt{c})}{8a^{7/8}c^{5/8}\sqrt{4-2\sqrt{2}}}
 \end{aligned}$$

$$\begin{aligned}
& + \frac{\arctan\left(\frac{-2c^{1/8}x + a^{1/8}\sqrt{2+\sqrt{2}}}{a^{1/8}\sqrt{2-\sqrt{2}}}\right) (-e\sqrt{a} + d(1-\sqrt{2})\sqrt{c})\sqrt{2+\sqrt{2}}}{8a^{7/8}c^{5/8}} \\
& - \frac{\arctan\left(\frac{2c^{1/8}x + a^{1/8}\sqrt{2+\sqrt{2}}}{a^{1/8}\sqrt{2-\sqrt{2}}}\right) (-e\sqrt{a} + d(1-\sqrt{2})\sqrt{c})\sqrt{2+\sqrt{2}}}{8a^{7/8}c^{5/8}} \\
& + \frac{\ln\left(a^{1/4} + c^{1/4}x^2 + c^{1/8}a^{1/8}\sqrt{2+\sqrt{2}}x\right) \left(d + d\sqrt{2} - \frac{e\sqrt{a}}{\sqrt{c}}\right)}{8a^{7/8}c^{1/8}\sqrt{4+2\sqrt{2}}} - \frac{\ln\left(a^{1/4} + c^{1/4}x^2 - c^{1/8}a^{1/8}\sqrt{2+\sqrt{2}}x\right) (-e\sqrt{a} + d(1+\sqrt{2})\sqrt{c})}{8a^{7/8}c^{5/8}\sqrt{4+2\sqrt{2}}}
\end{aligned}$$

Result(type 7, 33 leaves):

$$\frac{\sum_{R=\text{RootOf}(cZ^8+a)} \frac{(_R^4 e + d) \ln(x - _R)}{_R^7}}{8c}$$

Problem 3: Result is not expressed in closed-form.

$$\int \frac{ex^4 + d}{e^2x^8 - x^4b + d^2} dx$$

Optimal(type 3, 261 leaves, 7 steps):

$$\begin{aligned}
& - \frac{\arctan\left(\frac{x\sqrt{2}\sqrt{e}}{\sqrt{\sqrt{-2de+b} - \sqrt{2de+b}}}\right) \sqrt{e}\sqrt{2}}{2\sqrt{-2de+b}\sqrt{\sqrt{-2de+b} - \sqrt{2de+b}}} - \frac{\operatorname{arctanh}\left(\frac{x\sqrt{2}\sqrt{e}}{\sqrt{\sqrt{-2de+b} - \sqrt{2de+b}}}\right) \sqrt{e}\sqrt{2}}{2\sqrt{-2de+b}\sqrt{\sqrt{-2de+b} - \sqrt{2de+b}}} - \frac{\arctan\left(\frac{x\sqrt{2}\sqrt{e}}{\sqrt{\sqrt{-2de+b} + \sqrt{2de+b}}}\right) \sqrt{e}\sqrt{2}}{2\sqrt{-2de+b}\sqrt{\sqrt{-2de+b} + \sqrt{2de+b}}} \\
& - \frac{\operatorname{arctanh}\left(\frac{x\sqrt{2}\sqrt{e}}{\sqrt{\sqrt{-2de+b} + \sqrt{2de+b}}}\right) \sqrt{e}\sqrt{2}}{2\sqrt{-2de+b}\sqrt{\sqrt{-2de+b} + \sqrt{2de+b}}}
\end{aligned}$$

Result(type 7, 54 leaves):

$$\frac{\left(\sum_{R=\text{RootOf}(e^2Z^8-bZ^4+d^2)} \frac{(_R^4 e + d) \ln(x - _R)}{2_R^7 e^2 - _R^3 b}\right)}{4}$$

Problem 4: Result is not expressed in closed-form.

$$\int \frac{x^4 + 1}{x^8 + 3x^4 + 1} dx$$

Optimal (type 3, 293 leaves, 19 steps):

$$\begin{aligned} & \frac{\arctan\left(-1 + \frac{2^{3/4}x}{(3+\sqrt{5})^{1/4}}\right) (3-\sqrt{5})^{1/4} 2^{1/4}\sqrt{5}}{20} + \frac{\arctan\left(1 + \frac{2^{3/4}x}{(3+\sqrt{5})^{1/4}}\right) (3-\sqrt{5})^{1/4} 2^{1/4}\sqrt{5}}{20} \\ & - \frac{\ln\left(2x^2 - 22^{1/4}x(3+\sqrt{5})^{1/4} + \sqrt{5} + 1\right) (3-\sqrt{5})^{1/4} 2^{1/4}\sqrt{5}}{40} + \frac{\ln\left(2x^2 + 22^{1/4}x(3+\sqrt{5})^{1/4} + \sqrt{5} + 1\right) (3-\sqrt{5})^{1/4} 2^{1/4}\sqrt{5}}{40} \\ & + \frac{\arctan\left(-1 + \frac{2^{3/4}x}{(3-\sqrt{5})^{1/4}}\right) (3+\sqrt{5})^{1/4} 2^{1/4}\sqrt{5}}{20} + \frac{\arctan\left(1 + \frac{2^{3/4}x}{(3-\sqrt{5})^{1/4}}\right) (3+\sqrt{5})^{1/4} 2^{1/4}\sqrt{5}}{20} \\ & - \frac{\ln\left(2x^2 - 22^{1/4}x(3-\sqrt{5})^{1/4} + \sqrt{5} - 1\right) (3+\sqrt{5})^{1/4} 2^{1/4}\sqrt{5}}{40} + \frac{\ln\left(2x^2 + 22^{1/4}x(3-\sqrt{5})^{1/4} + \sqrt{5} - 1\right) (3+\sqrt{5})^{1/4} 2^{1/4}\sqrt{5}}{40} \end{aligned}$$

Result (type 7, 41 leaves):

$$\left(\frac{\sum_{R=\text{RootOf}(Z^8+3Z^4+1)} \frac{(-R^4+1)\ln(x-R)}{2R^7+3R^3}}{4} \right)$$

Problem 7: Result is not expressed in closed-form.

$$\int \frac{x^4 + 1}{x^8 - 4x^4 + 1} dx$$

Optimal (type 3, 101 leaves, 7 steps):

$$\frac{\arctan\left(\frac{2^{1/4}x}{\sqrt{\sqrt{3}-1}}\right) 2^{3/4}}{4\sqrt{\sqrt{3}-1}} + \frac{\operatorname{arctanh}\left(\frac{2^{1/4}x}{\sqrt{\sqrt{3}-1}}\right) 2^{3/4}}{4\sqrt{\sqrt{3}-1}} - \frac{\arctan\left(\frac{2^{1/4}x}{\sqrt{1+\sqrt{3}}}\right) 2^{3/4}}{4\sqrt{1+\sqrt{3}}} - \frac{\operatorname{arctanh}\left(\frac{2^{1/4}x}{\sqrt{1+\sqrt{3}}}\right) 2^{3/4}}{4\sqrt{1+\sqrt{3}}}$$

Result (type 7, 39 leaves):

$$\left(\frac{\sum_{R=\text{RootOf}(Z^8-4Z^4+1)} \frac{(-R^4+1)\ln(x-R)}{-R^7-2R^3}}{8} \right)$$

Problem 8: Result is not expressed in closed-form.

$$\int \frac{x^4 + 1}{x^8 - 5x^4 + 1} dx$$

Optimal (type 3, 123 leaves, 7 steps):

$$\frac{\arctan\left(\frac{x\sqrt{2}}{\sqrt{-\sqrt{3}+\sqrt{7}}}\right)}{\sqrt{-6\sqrt{3}+6\sqrt{7}}} + \frac{\operatorname{arctanh}\left(\frac{x\sqrt{2}}{\sqrt{-\sqrt{3}+\sqrt{7}}}\right)}{\sqrt{-6\sqrt{3}+6\sqrt{7}}} - \frac{\arctan\left(\frac{x\sqrt{2}}{\sqrt{\sqrt{3}+\sqrt{7}}}\right)}{\sqrt{6\sqrt{3}+6\sqrt{7}}} - \frac{\operatorname{arctanh}\left(\frac{x\sqrt{2}}{\sqrt{\sqrt{3}+\sqrt{7}}}\right)}{\sqrt{6\sqrt{3}+6\sqrt{7}}}$$

Result(type 7, 41 leaves):

$$\frac{\left(\sum_{R=\text{RootOf}(Z^8-5Z^4+1)} \frac{(-R^4+1)\ln(x-R)}{2R^7-5R^3}\right)}{4}$$

Problem 11: Result is not expressed in closed-form.

$$\int \frac{-x^4+1}{x^8-5x^4+1} dx$$

Optimal(type 3, 121 leaves, 7 steps):

$$\frac{\arctan\left(\frac{x\sqrt{2}}{\sqrt{-\sqrt{3}+\sqrt{7}}}\right)}{\sqrt{-14\sqrt{3}+14\sqrt{7}}} + \frac{\operatorname{arctanh}\left(\frac{x\sqrt{2}}{\sqrt{-\sqrt{3}+\sqrt{7}}}\right)}{\sqrt{-14\sqrt{3}+14\sqrt{7}}} + \frac{\arctan\left(\frac{x\sqrt{2}}{\sqrt{\sqrt{3}+\sqrt{7}}}\right)}{\sqrt{14\sqrt{3}+14\sqrt{7}}} + \frac{\operatorname{arctanh}\left(\frac{x\sqrt{2}}{\sqrt{\sqrt{3}+\sqrt{7}}}\right)}{\sqrt{14\sqrt{3}+14\sqrt{7}}}$$

Result(type 7, 43 leaves):

$$\frac{\left(\sum_{R=\text{RootOf}(Z^8-5Z^4+1)} \frac{(-R^4+1)\ln(x-R)}{2R^7-5R^3}\right)}{4}$$

Problem 13: Result is not expressed in closed-form.

$$\int \frac{d + \frac{e}{x^3}}{c + \frac{a}{x^6} + \frac{b}{x^3}} dx$$

Optimal(type 3, 577 leaves, 15 steps):

$$\frac{dx}{c} - \frac{\ln\left(2^{1/3}c^{1/3}x + (b - \sqrt{-4ac+b^2})^{1/3}\right) \left(bd - ce + \frac{2dca - b^2d + bce}{\sqrt{-4ac+b^2}}\right) 2^{2/3}}{6c^4/3 (b - \sqrt{-4ac+b^2})^{2/3}} + \frac{\ln\left(2^{2/3}c^{2/3}x^2 - 2^{1/3}c^{1/3}x(b - \sqrt{-4ac+b^2})^{1/3} + (b - \sqrt{-4ac+b^2})^{2/3}\right) \left(bd - ce + \frac{2dca - b^2d + bce}{\sqrt{-4ac+b^2}}\right) 2^{2/3}}{12c^4/3 (b - \sqrt{-4ac+b^2})^{2/3}}$$

$$\begin{aligned}
& + \frac{\arctan\left(\frac{\left(1 - \frac{2 \cdot 2^{1/3} c^{1/3} x}{(b - \sqrt{-4ac + b^2})^{1/3}}\right) \sqrt{3}}{3}\right) \left(bd - ce + \frac{2dca - b^2d + bce}{\sqrt{-4ac + b^2}}\right) 2^{2/3} \sqrt{3}}{6c^{4/3} (b - \sqrt{-4ac + b^2})^{2/3}} \\
& - \frac{\ln\left(2^{1/3} c^{1/3} x + (b + \sqrt{-4ac + b^2})^{1/3}\right) \left(bd - ce + \frac{-2dca + b^2d - bce}{\sqrt{-4ac + b^2}}\right) 2^{2/3}}{6c^{4/3} (b + \sqrt{-4ac + b^2})^{2/3}} \\
& + \frac{\ln\left(2^{2/3} c^{2/3} x^2 - 2^{1/3} c^{1/3} x (b + \sqrt{-4ac + b^2})^{1/3} + (b + \sqrt{-4ac + b^2})^{2/3}\right) \left(bd - ce + \frac{-2dca + b^2d - bce}{\sqrt{-4ac + b^2}}\right) 2^{2/3}}{12c^{4/3} (b + \sqrt{-4ac + b^2})^{2/3}} \\
& + \frac{\arctan\left(\frac{\left(1 - \frac{2 \cdot 2^{1/3} c^{1/3} x}{(b + \sqrt{-4ac + b^2})^{1/3}}\right) \sqrt{3}}{3}\right) \left(bd - ce + \frac{-2dca + b^2d - bce}{\sqrt{-4ac + b^2}}\right) 2^{2/3} \sqrt{3}}{6c^{4/3} (b + \sqrt{-4ac + b^2})^{2/3}}
\end{aligned}$$

Result(type 7, 66 leaves):

$$\frac{dx}{c} + \sum_{R=\text{RootOf}(Z^6c + Z^3b+a)} \frac{((-bd+ce)R^3 - ad) \ln(x - R)}{2R^5c + R^2b}$$

Problem 14: Result is not expressed in closed-form.

$$\int \frac{d + \frac{e}{x^4}}{c + \frac{a}{x^8} + \frac{b}{x^4}} dx$$

Optimal(type 3, 351 leaves, 9 steps):

$$\begin{aligned}
& \frac{dx}{c} + \frac{\arctan\left(\frac{2^{1/4} c^{1/4} x}{(-b - \sqrt{-4ac + b^2})^{1/4}}\right) \left(bd - ce + \frac{-2dca + b^2d - bce}{\sqrt{-4ac + b^2}}\right) 2^{3/4}}{4c^{5/4} (-b - \sqrt{-4ac + b^2})^{3/4}} \\
& + \frac{\operatorname{arctanh}\left(\frac{2^{1/4} c^{1/4} x}{(-b - \sqrt{-4ac + b^2})^{1/4}}\right) \left(bd - ce + \frac{-2dca + b^2d - bce}{\sqrt{-4ac + b^2}}\right) 2^{3/4}}{4c^{5/4} (-b - \sqrt{-4ac + b^2})^{3/4}}
\end{aligned}$$

$$\begin{aligned}
& + \frac{\arctan\left(\frac{2^{1/4} c^{1/4} x}{(-b + \sqrt{-4ac + b^2})^{1/4}}\right) \left(bd - ce + \frac{2dca - b^2d + bce}{\sqrt{-4ac + b^2}}\right) 2^{3/4}}{4c^{5/4} (-b + \sqrt{-4ac + b^2})^{3/4}} \\
& + \frac{\operatorname{arctanh}\left(\frac{2^{1/4} c^{1/4} x}{(-b + \sqrt{-4ac + b^2})^{1/4}}\right) \left(bd - ce + \frac{2dca - b^2d + bce}{\sqrt{-4ac + b^2}}\right) 2^{3/4}}{4c^{5/4} (-b + \sqrt{-4ac + b^2})^{3/4}}
\end{aligned}$$

Result(type 7, 66 leaves):

$$\frac{dx}{c} + \frac{\sum_{R=\text{RootOf}(cZ^8+bZ^4+a)} \frac{((-bd+ce)R^4-ad)\ln(x-R)}{2R^7c+R^3b}}{4c}$$

Problem 15: Unable to integrate problem.

$$\int \frac{1}{(d+ex^n)(a+cx^{2n})} dx$$

Optimal(type 5, 150 leaves, 6 steps):

$$\frac{cdx \operatorname{hypergeom}\left(\left[1, \frac{1}{2n}\right], \left[1 + \frac{1}{2n}\right], -\frac{cx^{2n}}{a}\right)}{a(ae^2+cd^2)} + \frac{e^2x \operatorname{hypergeom}\left(\left[1, \frac{1}{n}\right], \left[1 + \frac{1}{n}\right], -\frac{ex^n}{d}\right)}{d(ae^2+cd^2)} - \frac{ce^{x^{1+n}} \operatorname{hypergeom}\left(\left[1, \frac{1+n}{2n}\right], \left[\frac{3}{2} + \frac{1}{2n}\right], -\frac{cx^{2n}}{a}\right)}{a(ae^2+cd^2)(1+n)}$$

Result(type 8, 23 leaves):

$$\int \frac{1}{(d+ex^n)(a+cx^{2n})} dx$$

Problem 16: Unable to integrate problem.

$$\int \frac{d+ex^n}{a-cx^{2n}} dx$$

Optimal(type 5, 77 leaves, 3 steps):

$$\frac{dx \operatorname{hypergeom}\left(\left[1, \frac{1}{2n}\right], \left[1 + \frac{1}{2n}\right], \frac{cx^{2n}}{a}\right)}{a} + \frac{e^{x^{1+n}} \operatorname{hypergeom}\left(\left[1, \frac{1+n}{2n}\right], \left[\frac{3}{2} + \frac{1}{2n}\right], \frac{cx^{2n}}{a}\right)}{a(1+n)}$$

Result(type 8, 22 leaves):

$$\int \frac{d+ex^n}{a-cx^{2n}} dx$$

Problem 17: Unable to integrate problem.

$$\int \frac{(d + ex^n)^2}{(a + cx^{2n})^2} dx$$

Optimal(type 5, 193 leaves, 7 steps):

$$\frac{x(cd^2 - ae^2 + 2cde x^n)}{2acn(a + cx^{2n})} + \frac{e^2 x \operatorname{hypergeom}\left(\left[1, \frac{1}{2n}\right], \left[1 + \frac{1}{2n}\right], -\frac{cx^{2n}}{a}\right)}{ac} - \frac{(-ae^2 + cd^2)(1 - 2n)x \operatorname{hypergeom}\left(\left[1, \frac{1}{2n}\right], \left[1 + \frac{1}{2n}\right], -\frac{cx^{2n}}{a}\right)}{2a^2cn}$$

$$- \frac{de(1 - n)x^{1+n} \operatorname{hypergeom}\left(\left[1, \frac{1+n}{2n}\right], \left[\frac{3}{2} + \frac{1}{2n}\right], -\frac{cx^{2n}}{a}\right)}{a^2n(1+n)}$$

Result(type 8, 114 leaves):

$$- \frac{x(-2dee^{n \ln(x)}c + ae^2 - cd^2)}{2acn(a + c(e^{n \ln(x)})^2)} + \int \frac{2dee^{n \ln(x)}cn + 2cd^2n - 2dee^{n \ln(x)}c + ae^2 - cd^2}{2acn(a + c(e^{n \ln(x)})^2)} dx$$

Problem 18: Unable to integrate problem.

$$\int \frac{1}{(d + ex^n)(a + cx^{2n})^3} dx$$

Optimal(type 5, 558 leaves, 15 steps):

$$\frac{cx(d - ex^n)}{4a(ae^2 + cd^2)n(a + cx^{2n})^2} + \frac{c^2x(d - ex^n)}{2a(ae^2 + cd^2)^2n(a + cx^{2n})} - \frac{cx(d(1 - 4n) - e(1 - 3n)x^n)}{8a^2(ae^2 + cd^2)n^2(a + cx^{2n})}$$

$$+ \frac{cde^4x \operatorname{hypergeom}\left(\left[1, \frac{1}{2n}\right], \left[1 + \frac{1}{2n}\right], -\frac{cx^{2n}}{a}\right)}{a(ae^2 + cd^2)^3} + \frac{cd(1 - 4n)(1 - 2n)x \operatorname{hypergeom}\left(\left[1, \frac{1}{2n}\right], \left[1 + \frac{1}{2n}\right], -\frac{cx^{2n}}{a}\right)}{8a^3(ae^2 + cd^2)n^2}$$

$$- \frac{cde^2(1 - 2n)x \operatorname{hypergeom}\left(\left[1, \frac{1}{2n}\right], \left[1 + \frac{1}{2n}\right], -\frac{cx^{2n}}{a}\right)}{2a^2(ae^2 + cd^2)^2n} + \frac{e^6x \operatorname{hypergeom}\left(\left[1, \frac{1}{n}\right], \left[1 + \frac{1}{n}\right], -\frac{ex^n}{d}\right)}{d(ae^2 + cd^2)^3}$$

$$- \frac{ce^5x^{1+n} \operatorname{hypergeom}\left(\left[1, \frac{1+n}{2n}\right], \left[\frac{3}{2} + \frac{1}{2n}\right], -\frac{cx^{2n}}{a}\right)}{a(ae^2 + cd^2)^3(1+n)} - \frac{ce(1 - 3n)(1 - n)x^{1+n} \operatorname{hypergeom}\left(\left[1, \frac{1+n}{2n}\right], \left[\frac{3}{2} + \frac{1}{2n}\right], -\frac{cx^{2n}}{a}\right)}{8a^3(ae^2 + cd^2)n^2(1+n)}$$

$$+ \frac{ce^3(1 - n)x^{1+n} \operatorname{hypergeom}\left(\left[1, \frac{1+n}{2n}\right], \left[\frac{3}{2} + \frac{1}{2n}\right], -\frac{cx^{2n}}{a}\right)}{2a^2(ae^2 + cd^2)^2n(1+n)}$$

Result(type 8, 531 leaves):

$$\frac{1}{8a^2n^2(ae^2 + cd^2)^2(a + c(e^{n \ln(x)})^2)^2} (cx(-7ace^3n(e^{n \ln(x)})^3 - 3c^2d^2en(e^{n \ln(x)})^3 + 8acd^2n(e^{n \ln(x)})^2 + ace^3(e^{n \ln(x)})^3 + 4c^2d^3n(e^{n \ln(x)})^2$$

$$+ c^2d^2e(e^{n \ln(x)})^3 - 9a^2e^3n e^{n \ln(x)} - 5acd^2en e^{n \ln(x)} - acd^2e^2(e^{n \ln(x)})^2 - c^2(e^{n \ln(x)})^2d^3 + 10a^2de^2n + a^2e^3e^{n \ln(x)} + 6acd^3n + acd^2ee^{n \ln(x)})^2$$

$$\begin{aligned}
& -a^2 d e^2 - a c d^3) + \int \frac{1}{8 n^2 a^2 (d + e e^{n \ln(x)}) (a e^2 + c d^2)^2 (a + c (e^{n \ln(x)})^2)} \\
& (-7 a c e^4 n^2 (e^{n \ln(x)})^2 - 3 c^2 d^2 e^2 n^2 (e^{n \ln(x)})^2 + 9 a c d e^3 n^2 e^{n \ln(x)}) \\
& + 8 a c e^4 n (e^{n \ln(x)})^2 + 5 c^2 d^3 e n^2 e^{n \ln(x)} + 4 c^2 d^2 e^2 n (e^{n \ln(x)})^2 + 8 a^2 e^4 n^2 + 16 a c d^2 e^2 n^2 - 2 a c d e^3 n e^{n \ln(x)} - a c e^4 (e^{n \ln(x)})^2 + 8 c^2 d^4 n^2 \\
& - 2 c^2 d^3 e n e^{n \ln(x)} - c^2 d^2 e^2 (e^{n \ln(x)})^2 - 10 a c d^2 e^2 n - 6 c^2 d^4 n + a c d^2 e^2 + c^2 d^4) dx
\end{aligned}$$

Problem 20: Unable to integrate problem.

$$\int \frac{(a + c x^{2n})^p}{(d + e x^n)^2} dx$$

Optimal(type 6, 249 leaves, 8 steps):

$$\begin{aligned}
& \frac{e^2 x^{1+2n} (a + c x^{2n})^p \operatorname{AppellF1}\left(1 + \frac{1}{2n}, 2, -p, 2 + \frac{1}{2n}, \frac{e^2 x^{2n}}{d^2}, -\frac{c x^{2n}}{a}\right)}{d^4 (1 + 2n) \left(1 + \frac{c x^{2n}}{a}\right)^p} + \frac{x (a + c x^{2n})^p \operatorname{AppellF1}\left(\frac{1}{2n}, 2, -p, 1 + \frac{1}{2n}, \frac{e^2 x^{2n}}{d^2}, -\frac{c x^{2n}}{a}\right)}{d^2 \left(1 + \frac{c x^{2n}}{a}\right)^p} \\
& - \frac{2 e x^{1+n} (a + c x^{2n})^p \operatorname{AppellF1}\left(\frac{1+n}{2n}, 2, -p, \frac{3}{2} + \frac{1}{2n}, \frac{e^2 x^{2n}}{d^2}, -\frac{c x^{2n}}{a}\right)}{d^3 (1+n) \left(1 + \frac{c x^{2n}}{a}\right)^p}
\end{aligned}$$

Result(type 8, 23 leaves):

$$\int \frac{(a + c x^{2n})^p}{(d + e x^n)^2} dx$$

Problem 21: Unable to integrate problem.

$$\int \frac{(a + c x^{2n})^p}{(d + e x^n)^3} dx$$

Optimal(type 6, 341 leaves, 10 steps):

$$\begin{aligned}
& \frac{3 e^2 x^{1+2n} (a + c x^{2n})^p \operatorname{AppellF1}\left(1 + \frac{1}{2n}, 3, -p, 2 + \frac{1}{2n}, \frac{e^2 x^{2n}}{d^2}, -\frac{c x^{2n}}{a}\right)}{d^5 (1 + 2n) \left(1 + \frac{c x^{2n}}{a}\right)^p} \\
& - \frac{e^3 x^{1+3n} (a + c x^{2n})^p \operatorname{AppellF1}\left(\frac{3}{2} + \frac{1}{2n}, 3, -p, \frac{5}{2} + \frac{1}{2n}, \frac{e^2 x^{2n}}{d^2}, -\frac{c x^{2n}}{a}\right)}{d^6 (1 + 3n) \left(1 + \frac{c x^{2n}}{a}\right)^p} + \frac{x (a + c x^{2n})^p \operatorname{AppellF1}\left(\frac{1}{2n}, 3, -p, 1 + \frac{1}{2n}, \frac{e^2 x^{2n}}{d^2}, -\frac{c x^{2n}}{a}\right)}{d^3 \left(1 + \frac{c x^{2n}}{a}\right)^p}
\end{aligned}$$

$$- \frac{3 e x^{1+n} (a + c x^{2n})^p \operatorname{AppellF1}\left(\frac{1+n}{2n}, 3, -p, \frac{3}{2} + \frac{1}{2n}, \frac{e^2 x^{2n}}{d^2}, -\frac{c x^{2n}}{a}\right)}{d^4 (1+n) \left(1 + \frac{c x^{2n}}{a}\right)^p}$$

Result(type 8, 23 leaves):

$$\int \frac{(a + c x^{2n})^p}{(d + e x^n)^3} dx$$

Problem 23: Unable to integrate problem.

$$\int \frac{(d + e x^n)^2}{a + b x^n + c x^{2n}} dx$$

Optimal(type 5, 216 leaves, 5 steps):

$$\frac{e^2 x}{c} + \frac{x \operatorname{hypergeom}\left(\left[1, \frac{1}{n}\right], \left[1 + \frac{1}{n}\right], -\frac{2 c x^n}{b - \sqrt{-4 a c + b^2}}\right) \left(2 c d e - b e^2 + \frac{2 e^2 d^2 + b^2 e^2 - 2 c e (a e + b d)}{\sqrt{-4 a c + b^2}}\right)}{c (b - \sqrt{-4 a c + b^2})}$$

$$+ \frac{x \operatorname{hypergeom}\left(\left[1, \frac{1}{n}\right], \left[1 + \frac{1}{n}\right], -\frac{2 c x^n}{b + \sqrt{-4 a c + b^2}}\right) \left(2 c d e - b e^2 + \frac{-2 e^2 d^2 - b^2 e^2 + 2 c e (a e + b d)}{\sqrt{-4 a c + b^2}}\right)}{c (b + \sqrt{-4 a c + b^2})}$$

Result(type 8, 68 leaves):

$$\frac{e^2 x}{c} + \int -\frac{b e^2 e^{n \ln(x)} - 2 d e e^{n \ln(x)} c + a e^2 - c d^2}{c (a + b e^{n \ln(x)} + c (e^{n \ln(x)})^2)} dx$$

Problem 24: Unable to integrate problem.

$$\int \frac{(d + e x^n)^3}{(a + b x^n + c x^{2n})^3} dx$$

Optimal(type 5, 1674 leaves, 11 steps):

$$\frac{x (b^2 c d^3 - 2 a c d (-3 a e^2 + c d^2) - a b e (a e^2 + 3 c d^2) - (a b^2 e^3 + 2 a c e (-a e^2 + 3 c d^2) - b c d (3 a e^2 + c d^2)) x^n)}{2 a c (-4 a c + b^2) n (a + b x^n + c x^{2n})^2}$$

$$+ \frac{e^2 x (3 b^2 c d - 6 a c^2 d - b^3 e + a b c e + c (-2 a c e - b^2 e + 3 b c d) x^n)}{a c^2 (-4 a c + b^2) n (a + b x^n + c x^{2n})} - \frac{1}{2 a^2 e^2 (-4 a c + b^2)^2 n^2 (a + b x^n + c x^{2n})} (x (a b^2 c^2 d (3 a e^2 (1$$

$$- 9 n) - 5 c d^2 (1 - 3 n)) + 4 a^2 c^3 d (-3 a e^2 + c d^2) (1 - 4 n) - 2 a b^5 e^3 n + 2 a^2 b c^2 e (3 c d^2 (2 - 3 n) - 5 a e^2 n) - 3 a b^3 c e (-3 a e^2 n + c d^2)$$

$$+ b^4 c d (c d^2 (1 - 2 n) + 6 a e^2 n) + c (4 a^2 c^2 e (-a e^2 + 3 c d^2) (1 - 3 n) - 2 a b^4 e^3 n - 2 a b c^2 d (c d^2 (2 - 7 n) + 3 a e^2 n) + b^3 c d (c d^2 (1 - 2 n)$$

$$\begin{aligned}
& + 6ae^2n) - ab^2ce(3cd^2 - ae^2(1+2n))x^n) + \frac{1}{2a^2c(-4ac+b^2)^2n^2(b-\sqrt{-4ac+b^2})} \left(x \text{hypergeom} \left(\left[1, \frac{1}{n} \right], \left[1 + \frac{1}{n} \right], \right. \right. \\
& \left. \left. - \frac{2cx^n}{b-\sqrt{-4ac+b^2}} \right) \left((1-n)(4a^2c^2e(-ae^2+3cd^2)(1-3n) - 2ab^4e^3n - 2abc^2d(cd^2(2-7n) + 3ae^2n) + b^3cd(cd^2(1-2n) + 6ae^2n) \right. \right. \\
& \left. \left. - ab^2ce(3cd^2 - ae^2(1+2n))) + \frac{1}{\sqrt{-4ac+b^2}} (-2ab^5e^3(1-n)n + b^4cd(1-n)(cd^2(1-2n) + 6ae^2n) + 8a^2c^3d(-3ae^2 \right. \right. \\
& \left. \left. + cd^2)(8n^2 - 6n + 1) - 6ab^2c^2d(cd^2(3n^2 - 4n + 1) - ae^2(15n^2 - 10n + 1)) + 4a^2bc^2e(3cd^2(-3n^2 - n + 1) + ae^2(19n^2 - 11n + 1)) \right. \right. \\
& \left. \left. - ab^3ce(3cd^2(1-n) + ae^2(30n^2 - 19n + 1))) \right) + \frac{1}{2a^2c(-4ac+b^2)^2n^2(b+\sqrt{-4ac+b^2})} \left(x \text{hypergeom} \left(\left[1, \frac{1}{n} \right], \left[1 + \frac{1}{n} \right], \right. \right. \\
& \left. \left. - \frac{2cx^n}{b+\sqrt{-4ac+b^2}} \right) \left((1-n)(4a^2c^2e(-ae^2+3cd^2)(1-3n) - 2ab^4e^3n - 2abc^2d(cd^2(2-7n) + 3ae^2n) + b^3cd(cd^2(1-2n) + 6ae^2n) \right. \right. \\
& \left. \left. - ab^2ce(3cd^2 - ae^2(1+2n))) + \frac{1}{\sqrt{-4ac+b^2}} (2ab^5e^3(1-n)n - b^4cd(1-n)(cd^2(1-2n) + 6ae^2n) - 8a^2c^3d(-3ae^2 \right. \right. \\
& \left. \left. + cd^2)(8n^2 - 6n + 1) + 6ab^2c^2d(cd^2(3n^2 - 4n + 1) - ae^2(15n^2 - 10n + 1)) - 4a^2bc^2e(3cd^2(-3n^2 - n + 1) + ae^2(19n^2 - 11n + 1)) \right. \right. \\
& \left. \left. + ab^3ce(3cd^2(1-n) + ae^2(30n^2 - 19n + 1))) \right) + \frac{1}{ac(-4ac+b^2)n(b^2-4ac+b\sqrt{-4ac+b^2})} \left(e^2 x \text{hypergeom} \left(\left[1, \frac{1}{n} \right], \left[1 + \frac{1}{n} \right], \right. \right. \\
& \left. \left. - \frac{2cx^n}{b+\sqrt{-4ac+b^2}} \right) \left(-b^3e(1-n) + b^2(1-n)(3cd + e\sqrt{-4ac+b^2}) + bc(2ae(2-5n) - 3d(1-n)\sqrt{-4ac+b^2}) - 2ac(6cd(1-2n) \right. \right. \\
& \left. \left. - e(1-n)\sqrt{-4ac+b^2}) \right) \right) + \frac{1}{ac(-4ac+b^2)n(b^2-4ac-b\sqrt{-4ac+b^2})} \left(e^2 x \text{hypergeom} \left(\left[1, \frac{1}{n} \right], \left[1 + \frac{1}{n} \right], -\frac{2cx^n}{b-\sqrt{-4ac+b^2}} \right) \left(\right. \right. \\
& \left. \left. -b^3e(1-n) + b^2(1-n)(3cd - e\sqrt{-4ac+b^2}) + bc(2ae(2-5n) + 3d(1-n)\sqrt{-4ac+b^2}) - 2ac(6cd(1-2n) + e(1 \right. \right. \\
& \left. \left. - n)\sqrt{-4ac+b^2}) \right) \right)
\end{aligned}$$

Result(type 8, 1579 leaves):

$$\begin{aligned}
& \frac{1}{2(4ac-b^2)^2a^2n^2(a+be^{n \ln(x)}+c(e^{n \ln(x)})^2)^2} \left(x \left(-b^5d^3e^{n \ln(x)} - 4a^3c^2d^3 - 3a^3b^2de^2n + 30a^3bcd^2en - 3a^2b^3d^2en + 4a^3c^2e^3(e^{n \ln(x)})^3 \right. \right. \\
& \left. \left. - b^3c^2d^3(e^{n \ln(x)})^3 - a^2b^3e^3(e^{n \ln(x)})^2 - 4a^2c^3d^3(e^{n \ln(x)})^2 + 2b^5d^3ne^{n \ln(x)} - 2b^4cd^3(e^{n \ln(x)})^2 + 4a^4c^3e^{n \ln(x)} - a^3b^2e^3e^{n \ln(x)} + 24a^3c^2d^3n \right. \right. \\
& \left. \left. + 12a^4cde^2 + 5a^2b^2cd^3 - ab^4d^3 - 12a^2c^3d^2e(e^{n \ln(x)})^3 + 4ab^3c^3d^3(e^{n \ln(x)})^3 + 4b^4cd^3n(e^{n \ln(x)})^2 - 4a^4ce^3ne^{n \ln(x)} + 10a^3b^2e^3ne^{n \ln(x)} \right. \right. \\
& \left. \left. + 4a^3bc^3(e^{n \ln(x)})^2 + 12a^3c^2de^2(e^{n \ln(x)})^2 + 9ab^2c^2d^3(e^{n \ln(x)})^2 - 12a^3c^2d^2e^{n \ln(x)} - 3a^2b^3de^2e^{n \ln(x)} + 3ab^4d^2e^{n \ln(x)} + 4ab^3cd^3e^{n \ln(x)} \right. \right. \\
& \left. \left. - 18a^2b^2d^2e^2n(e^{n \ln(x)})^3 - 27a^2b^2cd^2e^{n \ln(x)}(e^{n \ln(x)})^2 + 54a^2b^2d^2en(e^{n \ln(x)})^2 - 30a^3bcd^2en e^{n \ln(x)} + 12a^2b^2cd^2en e^{n \ln(x)} + 2a^2b^2ce^3n(e^{n \ln(x)})^3 \right. \right. \\
& \left. \left. + 36a^2c^3d^2en(e^{n \ln(x)})^3 - 14ab^3c^3d^3n(e^{n \ln(x)})^3 + 6a^3bc^3n(e^{n \ln(x)})^2 - 29a^2b^2c^2d^3n(e^{n \ln(x)})^2 + 3ab^2c^2d^2e(e^{n \ln(x)})^3 + 60a^3c^2d^2en e^{n \ln(x)} \right. \right. \\
& \left. \left. - 6a^2b^3d^2ne^{n \ln(x)} - 3a^2b^2cd^2e(e^{n \ln(x)})^2 - 2a^2bc^2d^3ne^{n \ln(x)} - 24a^2bc^2d^2e(e^{n \ln(x)})^2 - 12ab^3cd^3ne^{n \ln(x)} + 6ab^3cd^2e(e^{n \ln(x)})^2 \right) \right)
\end{aligned}$$

$$\begin{aligned}
& + 12a^3bcd e^2 e^{n \ln(x)} - 9a^2b^2cd^2 e e^{n \ln(x)} + 4a^3c^2e^3 n (e^{n \ln(x)})^3 + 2b^3c^2d^3 n (e^{n \ln(x)})^3 + 3a^2b^3e^3 n (e^{n \ln(x)})^2 - a^2b^2c^3 (e^{n \ln(x)})^3 \\
& + 16a^2c^3d^3 n (e^{n \ln(x)})^2 - 21a^2b^2cd^3 n + 3ab^4d^3 n - 3a^3b^2d^2 e - 12a^3bcd^2 e + 3a^2b^3d^2 e + 6a^4be^3 n - 24a^4cd^2 n) + \int \\
& - \frac{1}{2(4ac - b^2)^2 a^2 n^2 (a + b e^{n \ln(x)} + c (e^{n \ln(x)})^2)} (-36a^2c^2d^2 e n^2 e^{n \ln(x)} + 14ab c^2 d^3 n^2 e^{n \ln(x)} + 48a^2c^2d^2 e n e^{n \ln(x)} - 18ab c^2 d^3 n e^{n \ln(x)} \\
& + 3ab^2cd^2 e e^{n \ln(x)} - 4a^2c^2d^3 - b^4d^3 + 4a^3c^3 e^{n \ln(x)} - a^2b^2e^3 e^{n \ln(x)} - b^3cd^3 e^{n \ln(x)} - 3ab^3d^2 e n - 3a^2b^2d^2 e n + 30a^2bcd^2 e n \\
& + 18a^2bcd e^2 n^2 e^{n \ln(x)} - 18a^2bcd e^2 n e^{n \ln(x)} - 3ab^2cd^2 e n e^{n \ln(x)} - 32a^2c^2d^3 n^2 - 2b^4d^3 n^2 + 5ab^2cd^3 + 24a^2c^2d^3 n + 3b^4d^3 n + 12a^3cd e^2 \\
& - 4a^3c^3n^2 e^{n \ln(x)} - 2a^2b^2e^3 n^2 e^{n \ln(x)} - 2b^3cd^3 n^2 e^{n \ln(x)} + 3a^2b^2e^3 n e^{n \ln(x)} + 3b^3cd^3 n e^{n \ln(x)} - 12a^2c^2d^2 e e^{n \ln(x)} + 4ab c^2 d^3 e^{n \ln(x)} + 16a^2cd^3 n^2 \\
& + 6a^3be^3 n - 21ab^2cd^3 n - 3a^2b^2d^2 e - 12a^2bcd^2 e - 24a^3cd^2 n + 3ab^3d^2 e) dx
\end{aligned}$$

Problem 25: Unable to integrate problem.

$$\int \frac{(d + ex^n)^2}{(a + bx^n + cx^{2n})^3} dx$$

Optimal (type 5, 1165 leaves, 11 steps):

$$\begin{aligned}
& \frac{x(b^2d^2 - 2abde - 2a(-ae^2 + cd^2) + (abe^2 - 4dcae + bcd^2)x^n)}{2a(-4ac + b^2)n(a + bx^n + cx^{2n})^2} + \frac{e^2x(b^2 - 2ac + bcx^n)}{ac(-4ac + b^2)n(a + bx^n + cx^{2n})} \\
& + \frac{1}{2a^2c(-4ac + b^2)^2n^2(a + bx^n + cx^{2n})} (x(2ab^3cde - ab^2c(ae^2(1 - 9n) - 5cd^2(1 - 3n)) - 4a^2c^2(-ae^2 + cd^2)(1 - 4n) - 4a^2bc^2de(2 \\
& - 3n) - b^4(cd^2(1 - 2n) + 2ae^2n) + c(2ab^2cde - 8a^2c^2de(1 - 3n) + 2abc(cd^2(2 - 7n) + ae^2n) - b^3(cd^2(1 - 2n) + 2ae^2n))x^n) \\
&) - \frac{1}{2a^2(-4ac + b^2)^2n^2(b - \sqrt{-4ac + b^2})} \left(x \operatorname{hypergeom} \left(\left[1, \frac{1}{n} \right], \left[1 + \frac{1}{n} \right], -\frac{2cx^n}{b - \sqrt{-4ac + b^2}} \right) \left((1 - n)(2ab^2cde - 8a^2c^2de(1 - 3n) \right. \right. \\
& + 2abc(cd^2(2 - 7n) + ae^2n) - b^3(cd^2(1 - 2n) + 2ae^2n)) + \frac{1}{\sqrt{-4ac + b^2}} (2ab^3cde(1 - n) - b^4(1 - n)(cd^2(1 - 2n) + 2ae^2n) \\
& - 8a^2bc^2de(-3n^2 - n + 1) - 8a^2c^2(-ae^2 + cd^2)(8n^2 - 6n + 1) + 2ab^2c(3cd^2(3n^2 - 4n + 1) - ae^2(15n^2 - 10n + 1))) \left. \right) \Bigg) \\
& - \frac{1}{2a^2(-4ac + b^2)^2n^2(b + \sqrt{-4ac + b^2})} \left(x \operatorname{hypergeom} \left(\left[1, \frac{1}{n} \right], \left[1 + \frac{1}{n} \right], -\frac{2cx^n}{b + \sqrt{-4ac + b^2}} \right) \left((1 - n)(2ab^2cde - 8a^2c^2de(1 - 3n) \right. \right. \\
& + 2abc(cd^2(2 - 7n) + ae^2n) - b^3(cd^2(1 - 2n) + 2ae^2n)) + \frac{1}{\sqrt{-4ac + b^2}} (-2ab^3cde(1 - n) + b^4(1 - n)(cd^2(1 - 2n) + 2ae^2n) \\
& + 8a^2bc^2de(-3n^2 - n + 1) + 8a^2c^2(-ae^2 + cd^2)(8n^2 - 6n + 1) - 2ab^2c(3cd^2(3n^2 - 4n + 1) - ae^2(15n^2 - 10n + 1))) \left. \right) \Bigg)
\end{aligned}$$

$$\frac{e^2 x \text{hypergeom}\left(\left[1, \frac{1}{n}\right], \left[1 + \frac{1}{n}\right], -\frac{2cx^n}{b - \sqrt{-4ac + b^2}}\right) \left(4ac(1-2n) - b^2(1-n) - b(1-n)\sqrt{-4ac + b^2}\right)}{a(-4ac + b^2)n(b^2 - 4ac - b\sqrt{-4ac + b^2})}$$

$$\frac{e^2 x \text{hypergeom}\left(\left[1, \frac{1}{n}\right], \left[1 + \frac{1}{n}\right], -\frac{2cx^n}{b + \sqrt{-4ac + b^2}}\right) \left(4ac(1-2n) - b^2(1-n) + b(1-n)\sqrt{-4ac + b^2}\right)}{a(-4ac + b^2)n(b^2 - 4ac + b\sqrt{-4ac + b^2})}$$

Result(type 8, 1189 leaves):

$$\frac{1}{2(4ac - b^2)^2 a^2 n^2 (a + b e^{n \ln(x)} + c (e^{n \ln(x)})^2)^2} \left(x \left(-4a^4 c e^2 + 4d^2 c^2 a^3 + b^5 d^2 e^{n \ln(x)} - 36a^2 b c^2 d e n (e^{n \ln(x)})^2 - 8a^2 b^2 c d e n e^{n \ln(x)} - 20a^3 b c d e n \right. \right. \\ \left. \left. + 2a^2 b^3 d e n + 6a^2 b c^2 e^2 n (e^{n \ln(x)})^3 - 24a^2 c^3 d e n (e^{n \ln(x)})^3 + 14ab c^3 d^2 n (e^{n \ln(x)})^3 + 9a^2 b^2 c e^2 n (e^{n \ln(x)})^2 + 29a b^2 c^2 d^2 n (e^{n \ln(x)})^2 \right. \right. \\ \left. \left. - 2a b^2 c^2 d e (e^{n \ln(x)})^3 + 10a^3 b c e^2 n e^{n \ln(x)} - 40a^3 c^2 d e n e^{n \ln(x)} + 2a^2 b c^2 d^2 n e^{n \ln(x)} + 16a^2 b c^2 d e (e^{n \ln(x)})^2 + 12a b^3 c d^2 n e^{n \ln(x)} \right. \right. \\ \left. \left. - 4a b^3 c d e (e^{n \ln(x)})^2 + 6a^2 b^2 c d e e^{n \ln(x)} - 2b^3 c^2 d^2 n (e^{n \ln(x)})^3 - 16a^2 c^3 d^2 n (e^{n \ln(x)})^2 + 8a^2 c^3 d e (e^{n \ln(x)})^3 - 4a b c^3 d^2 (e^{n \ln(x)})^3 \right. \right. \\ \left. \left. - 4b^4 c d^2 n (e^{n \ln(x)})^2 + 2a^2 b^3 e^2 n e^{n \ln(x)} + a^2 b^2 c e^2 (e^{n \ln(x)})^2 - 9a b^2 c^2 d^2 (e^{n \ln(x)})^2 - 4a^3 b c e^2 e^{n \ln(x)} + 8a^3 c^2 d e e^{n \ln(x)} - 2a b^4 d e e^{n \ln(x)} \right. \right. \\ \left. \left. - 4a b^3 c d^2 e^{n \ln(x)} + a^3 b^2 e^2 n + 21a^2 b^2 c d^2 n - 3a b^4 d^2 n + 8a^3 b c d e - 2a^2 b^3 d e + b^3 c^2 d^2 (e^{n \ln(x)})^3 - 4a^3 c^2 e^2 (e^{n \ln(x)})^2 + 4a^2 c^3 d^2 (e^{n \ln(x)})^2 \right. \right. \\ \left. \left. - 2b^5 d^2 n e^{n \ln(x)} + 2b^4 c d^2 (e^{n \ln(x)})^2 + a^2 b^3 e^2 e^{n \ln(x)} + 8a^4 c e^2 n - 24a^3 c^2 d^2 n + a^3 b^2 e^2 - 5a^2 b^2 c d^2 + a b^4 d^2 \right) \right) + \\ \int \frac{1}{2(4ac - b^2)^2 a^2 n^2 (a + b e^{n \ln(x)} + c (e^{n \ln(x)})^2)} \left(-4a^3 c e^2 + b^4 d^2 + 4d^2 c^2 a^2 - 5a b^2 c d^2 + 2a b^2 c d e n e^{n \ln(x)} - 20a^2 b c d e n + 2a b^3 d e n \right. \\ \left. - 6a^2 b c e^2 n^2 e^{n \ln(x)} + 24a^2 c^2 d e n^2 e^{n \ln(x)} - 14a b c^2 d^2 n^2 e^{n \ln(x)} + 6a^2 b c e^2 n e^{n \ln(x)} - 32a^2 c^2 d e n e^{n \ln(x)} + 18a b c^2 d^2 n e^{n \ln(x)} - 2a b^2 c d e e^{n \ln(x)} \right. \\ \left. + 2b^3 c d^2 n^2 e^{n \ln(x)} - 3b^3 c d^2 n e^{n \ln(x)} + 8a^2 c^2 d e e^{n \ln(x)} - 4a b c^2 d^2 e^{n \ln(x)} - 2a b^3 d e + a^2 b^2 e^2 n + 21a b^2 c d^2 n + 8a^2 b c d e - 16a b^2 c d^2 n^2 \right. \\ \left. + b^3 c d^2 e^{n \ln(x)} + 32a^2 c^2 d^2 n^2 + 2b^4 d^2 n^2 + a^2 b^2 e^2 + 8a^3 c e^2 n - 24a^2 c^2 d^2 n - 3b^4 d^2 n \right) dx$$

Problem 26: Unable to integrate problem.

$$\int (d + ex^n)^3 (a + bx^n + cx^{2n})^p dx$$

Optimal(type 6, 574 leaves, 10 steps):

$$\frac{3d^2 ex^{1+n} (a + bx^n + cx^{2n})^p \text{AppellF1}\left(1 + \frac{1}{n}, -p, -p, 2 + \frac{1}{n}, -\frac{2cx^n}{b - \sqrt{-4ac + b^2}}, -\frac{2cx^n}{b + \sqrt{-4ac + b^2}}\right)}{(1+n) \left(1 + \frac{2cx^n}{b - \sqrt{-4ac + b^2}}\right)^p \left(1 + \frac{2cx^n}{b + \sqrt{-4ac + b^2}}\right)^p}$$

$$\begin{aligned}
& \frac{3 d e^2 x^{1+2n} (a + b x^n + c x^{2n})^p \operatorname{AppellF1}\left(2 + \frac{1}{n}, -p, -p, 3 + \frac{1}{n}, -\frac{2 c x^n}{b - \sqrt{-4 a c + b^2}}, -\frac{2 c x^n}{b + \sqrt{-4 a c + b^2}}\right)}{(1 + 2n) \left(1 + \frac{2 c x^n}{b - \sqrt{-4 a c + b^2}}\right)^p \left(1 + \frac{2 c x^n}{b + \sqrt{-4 a c + b^2}}\right)^p} \\
& + \frac{e^3 x^{1+3n} (a + b x^n + c x^{2n})^p \operatorname{AppellF1}\left(3 + \frac{1}{n}, -p, -p, 4 + \frac{1}{n}, -\frac{2 c x^n}{b - \sqrt{-4 a c + b^2}}, -\frac{2 c x^n}{b + \sqrt{-4 a c + b^2}}\right)}{(1 + 3n) \left(1 + \frac{2 c x^n}{b - \sqrt{-4 a c + b^2}}\right)^p \left(1 + \frac{2 c x^n}{b + \sqrt{-4 a c + b^2}}\right)^p} \\
& + \frac{d^3 x (a + b x^n + c x^{2n})^p \operatorname{AppellF1}\left(\frac{1}{n}, -p, -p, 1 + \frac{1}{n}, -\frac{2 c x^n}{b - \sqrt{-4 a c + b^2}}, -\frac{2 c x^n}{b + \sqrt{-4 a c + b^2}}\right)}{\left(1 + \frac{2 c x^n}{b - \sqrt{-4 a c + b^2}}\right)^p \left(1 + \frac{2 c x^n}{b + \sqrt{-4 a c + b^2}}\right)^p}
\end{aligned}$$

Result(type 8, 28 leaves):

$$\int (d + e x^n)^3 (a + b x^n + c x^{2n})^p dx$$

Problem 27: Unable to integrate problem.

$$\int (d + e x^n)^2 (a + b x^n + c x^{2n})^p dx$$

Optimal(type 6, 423 leaves, 8 steps):

$$\begin{aligned}
& \frac{2 d e x^{1+n} (a + b x^n + c x^{2n})^p \operatorname{AppellF1}\left(1 + \frac{1}{n}, -p, -p, 2 + \frac{1}{n}, -\frac{2 c x^n}{b - \sqrt{-4 a c + b^2}}, -\frac{2 c x^n}{b + \sqrt{-4 a c + b^2}}\right)}{(1 + n) \left(1 + \frac{2 c x^n}{b - \sqrt{-4 a c + b^2}}\right)^p \left(1 + \frac{2 c x^n}{b + \sqrt{-4 a c + b^2}}\right)^p} \\
& + \frac{e^2 x^{1+2n} (a + b x^n + c x^{2n})^p \operatorname{AppellF1}\left(2 + \frac{1}{n}, -p, -p, 3 + \frac{1}{n}, -\frac{2 c x^n}{b - \sqrt{-4 a c + b^2}}, -\frac{2 c x^n}{b + \sqrt{-4 a c + b^2}}\right)}{(1 + 2n) \left(1 + \frac{2 c x^n}{b - \sqrt{-4 a c + b^2}}\right)^p \left(1 + \frac{2 c x^n}{b + \sqrt{-4 a c + b^2}}\right)^p} \\
& + \frac{d^2 x (a + b x^n + c x^{2n})^p \operatorname{AppellF1}\left(\frac{1}{n}, -p, -p, 1 + \frac{1}{n}, -\frac{2 c x^n}{b - \sqrt{-4 a c + b^2}}, -\frac{2 c x^n}{b + \sqrt{-4 a c + b^2}}\right)}{\left(1 + \frac{2 c x^n}{b - \sqrt{-4 a c + b^2}}\right)^p \left(1 + \frac{2 c x^n}{b + \sqrt{-4 a c + b^2}}\right)^p}
\end{aligned}$$

Result(type 8, 28 leaves):

$$\int (d + ex^n)^2 (a + bx^n + cx^{2n})^p dx$$

Test results for the 46 problems in "1.2.3.4 (f x)^m (d+e x^n)^q (a+b x^n+c x^(2 n))^p.txt"

Problem 5: Result is not expressed in closed-form.

$$\int \frac{x^3 (ex^3 + d)}{cx^6 + bx^3 + a} dx$$

Optimal(type 3, 577 leaves, 14 steps):

$$\frac{ex}{c} + \frac{\ln\left(2^{1/3} c^{1/3} x + \left(b - \sqrt{-4ac + b^2}\right)^{1/3}\right) \left(cd - be + \frac{-2ace + b^2e - bcd}{\sqrt{-4ac + b^2}}\right) 2^{2/3}}{6c^4/3 \left(b - \sqrt{-4ac + b^2}\right)^{2/3}}$$

$$\frac{\ln\left(2^{2/3} c^{2/3} x^2 - 2^{1/3} c^{1/3} x \left(b - \sqrt{-4ac + b^2}\right)^{1/3} + \left(b - \sqrt{-4ac + b^2}\right)^{2/3}\right) \left(cd - be + \frac{-2ace + b^2e - bcd}{\sqrt{-4ac + b^2}}\right) 2^{2/3}}{12c^4/3 \left(b - \sqrt{-4ac + b^2}\right)^{2/3}}$$

$$\frac{\arctan\left(\frac{\left(1 - \frac{2 \cdot 2^{1/3} c^{1/3} x}{\left(b - \sqrt{-4ac + b^2}\right)^{1/3}}\right) \sqrt{3}}{3}\right) \left(cd - be + \frac{-2ace + b^2e - bcd}{\sqrt{-4ac + b^2}}\right) 2^{2/3} \sqrt{3}}{6c^4/3 \left(b - \sqrt{-4ac + b^2}\right)^{2/3}}$$

$$+ \frac{\ln\left(2^{1/3} c^{1/3} x + \left(b + \sqrt{-4ac + b^2}\right)^{1/3}\right) \left(cd - be + \frac{2ace - b^2e + bcd}{\sqrt{-4ac + b^2}}\right) 2^{2/3}}{6c^4/3 \left(b + \sqrt{-4ac + b^2}\right)^{2/3}}$$

$$\frac{\ln\left(2^{2/3} c^{2/3} x^2 - 2^{1/3} c^{1/3} x \left(b + \sqrt{-4ac + b^2}\right)^{1/3} + \left(b + \sqrt{-4ac + b^2}\right)^{2/3}\right) \left(cd - be + \frac{2ace - b^2e + bcd}{\sqrt{-4ac + b^2}}\right) 2^{2/3}}{12c^4/3 \left(b + \sqrt{-4ac + b^2}\right)^{2/3}}$$

$$\frac{\arctan\left(\frac{\left(1 - \frac{2 \cdot 2^{1/3} c^{1/3} x}{\left(b + \sqrt{-4ac + b^2}\right)^{1/3}}\right) \sqrt{3}}{3}\right) \left(cd - be + \frac{2ace - b^2e + bcd}{\sqrt{-4ac + b^2}}\right) 2^{2/3} \sqrt{3}}{6c^4/3 \left(b + \sqrt{-4ac + b^2}\right)^{2/3}}$$

Result(type 7, 66 leaves):

$$\frac{ex}{c} + \frac{\sum_{R=\text{RootOf}(Z^6 c + Z^3 b + a)} ((-be + cd) R^3 - ae) \ln(x - R)}{3c}$$

Problem 6: Result is not expressed in closed-form.

$$\int \frac{x(ex^3 + d)}{cx^6 + bx^3 + a} dx$$

Optimal (type 3, 490 leaves, 13 steps):

$$\begin{aligned} & \frac{\ln\left(2^{1/3} c^{1/3} x + (b - \sqrt{-4ac + b^2})^{1/3}\right) \left(e + \frac{-be + 2cd}{\sqrt{-4ac + b^2}}\right) 2^{1/3}}{6c^{2/3} (b - \sqrt{-4ac + b^2})^{1/3}} \\ & + \frac{\ln\left(2^{2/3} c^{2/3} x^2 - 2^{1/3} c^{1/3} x (b - \sqrt{-4ac + b^2})^{1/3} + (b - \sqrt{-4ac + b^2})^{2/3}\right) \left(e + \frac{-be + 2cd}{\sqrt{-4ac + b^2}}\right) 2^{1/3}}{12c^{2/3} (b - \sqrt{-4ac + b^2})^{1/3}} \\ & - \frac{\arctan\left(\frac{\left(1 - \frac{2 \cdot 2^{1/3} c^{1/3} x}{(b - \sqrt{-4ac + b^2})^{1/3}}\right) \sqrt{3}}{3}\right) \left(e + \frac{-be + 2cd}{\sqrt{-4ac + b^2}}\right) 2^{1/3} \sqrt{3}}{6c^{2/3} (b - \sqrt{-4ac + b^2})^{1/3}} \\ & - \frac{\ln\left(2^{1/3} c^{1/3} x + (b + \sqrt{-4ac + b^2})^{1/3}\right) \left(e + \frac{be - 2cd}{\sqrt{-4ac + b^2}}\right) 2^{1/3}}{6c^{2/3} (b + \sqrt{-4ac + b^2})^{1/3}} \\ & + \frac{\ln\left(2^{2/3} c^{2/3} x^2 - 2^{1/3} c^{1/3} x (b + \sqrt{-4ac + b^2})^{1/3} + (b + \sqrt{-4ac + b^2})^{2/3}\right) \left(e + \frac{be - 2cd}{\sqrt{-4ac + b^2}}\right) 2^{1/3}}{12c^{2/3} (b + \sqrt{-4ac + b^2})^{1/3}} \\ & - \frac{\arctan\left(\frac{\left(1 - \frac{2 \cdot 2^{1/3} c^{1/3} x}{(b + \sqrt{-4ac + b^2})^{1/3}}\right) \sqrt{3}}{3}\right) \left(e + \frac{be - 2cd}{\sqrt{-4ac + b^2}}\right) 2^{1/3} \sqrt{3}}{6c^{2/3} (b + \sqrt{-4ac + b^2})^{1/3}} \end{aligned}$$

Result (type 7, 48 leaves):

$$\frac{\left(\sum_{R=\text{RootOf}(Z^6 c + Z^3 b + a)} \frac{(R^4 e + R d) \ln(x - R)}{2 R^5 c + R^2 b} \right)}{3}$$

Problem 7: Result is not expressed in closed-form.

$$\int \frac{ex^3 + d}{cx^6 + bx^3 + a} dx$$

Optimal (type 3, 490 leaves, 13 steps):

$$\frac{\ln\left(2^{1/3} c^{1/3} x + (b - \sqrt{-4ac + b^2})^{1/3}\right) \left(e + \frac{-be + 2cd}{\sqrt{-4ac + b^2}}\right) 2^{2/3}}{6 c^{1/3} (b - \sqrt{-4ac + b^2})^{2/3}}$$

$$- \frac{\ln\left(2^{2/3} c^{2/3} x^2 - 2^{1/3} c^{1/3} x (b - \sqrt{-4ac + b^2})^{1/3} + (b - \sqrt{-4ac + b^2})^{2/3}\right) \left(e + \frac{-be + 2cd}{\sqrt{-4ac + b^2}}\right) 2^{2/3}}{12 c^{1/3} (b - \sqrt{-4ac + b^2})^{2/3}}$$

$$- \frac{\arctan\left(\frac{\left(1 - \frac{2 \cdot 2^{1/3} c^{1/3} x}{(b - \sqrt{-4ac + b^2})^{1/3}}\right) \sqrt{3}}{3}\right) \left(e + \frac{-be + 2cd}{\sqrt{-4ac + b^2}}\right) 2^{2/3} \sqrt{3}}{6 c^{1/3} (b - \sqrt{-4ac + b^2})^{2/3}}$$

$$+ \frac{\ln\left(2^{1/3} c^{1/3} x + (b + \sqrt{-4ac + b^2})^{1/3}\right) \left(e + \frac{be - 2cd}{\sqrt{-4ac + b^2}}\right) 2^{2/3}}{6 c^{1/3} (b + \sqrt{-4ac + b^2})^{2/3}}$$

$$- \frac{\ln\left(2^{2/3} c^{2/3} x^2 - 2^{1/3} c^{1/3} x (b + \sqrt{-4ac + b^2})^{1/3} + (b + \sqrt{-4ac + b^2})^{2/3}\right) \left(e + \frac{be - 2cd}{\sqrt{-4ac + b^2}}\right) 2^{2/3}}{12 c^{1/3} (b + \sqrt{-4ac + b^2})^{2/3}}$$

$$- \frac{\arctan\left(\frac{\left(1 - \frac{2 \cdot 2^{1/3} c^{1/3} x}{(b + \sqrt{-4ac + b^2})^{1/3}}\right) \sqrt{3}}{3}\right) \left(e + \frac{be - 2cd}{\sqrt{-4ac + b^2}}\right) 2^{2/3} \sqrt{3}}{6 c^{1/3} (b + \sqrt{-4ac + b^2})^{2/3}}$$

Result (type 7, 46 leaves):

$$\frac{\left(\sum_{R=\text{RootOf}(Z^6 c + Z^3 b + a)} \frac{(R^3 e + d) \ln(x - R)}{2 R^5 c + R^2 b} \right)}{3}$$

Problem 8: Result is not expressed in closed-form.

$$\int \frac{ex^3 + d}{x^2 (cx^6 + bx^3 + a)} dx$$

Optimal(type 3, 517 leaves, 14 steps):

$$\begin{aligned} & -\frac{d}{xa} + \frac{c^{1/3} \ln\left(2^{1/3} c^{1/3} x + (b - \sqrt{-4ac + b^2})^{1/3}\right) \left(d + \frac{-2ae + bd}{\sqrt{-4ac + b^2}}\right) 2^{1/3}}{6a (b - \sqrt{-4ac + b^2})^{1/3}} \\ & - \frac{c^{1/3} \ln\left(2^{2/3} c^{2/3} x^2 - 2^{1/3} c^{1/3} x (b - \sqrt{-4ac + b^2})^{1/3} + (b - \sqrt{-4ac + b^2})^{2/3}\right) \left(d + \frac{-2ae + bd}{\sqrt{-4ac + b^2}}\right) 2^{1/3}}{12a (b - \sqrt{-4ac + b^2})^{1/3}} \\ & + \frac{c^{1/3} \arctan\left(\frac{\left(1 - \frac{2 \cdot 2^{1/3} c^{1/3} x}{(b - \sqrt{-4ac + b^2})^{1/3}}\right) \sqrt{3}}{3}\right) \left(d + \frac{-2ae + bd}{\sqrt{-4ac + b^2}}\right) 2^{1/3} \sqrt{3}}{6a (b - \sqrt{-4ac + b^2})^{1/3}} \\ & + \frac{c^{1/3} \ln\left(2^{1/3} c^{1/3} x + (b + \sqrt{-4ac + b^2})^{1/3}\right) \left(d + \frac{2ae - bd}{\sqrt{-4ac + b^2}}\right) 2^{1/3}}{6a (b + \sqrt{-4ac + b^2})^{1/3}} \\ & - \frac{c^{1/3} \ln\left(2^{2/3} c^{2/3} x^2 - 2^{1/3} c^{1/3} x (b + \sqrt{-4ac + b^2})^{1/3} + (b + \sqrt{-4ac + b^2})^{2/3}\right) \left(d + \frac{2ae - bd}{\sqrt{-4ac + b^2}}\right) 2^{1/3}}{12a (b + \sqrt{-4ac + b^2})^{1/3}} \\ & + \frac{c^{1/3} \arctan\left(\frac{\left(1 - \frac{2 \cdot 2^{1/3} c^{1/3} x}{(b + \sqrt{-4ac + b^2})^{1/3}}\right) \sqrt{3}}{3}\right) \left(d + \frac{2ae - bd}{\sqrt{-4ac + b^2}}\right) 2^{1/3} \sqrt{3}}{6a (b + \sqrt{-4ac + b^2})^{1/3}} \end{aligned}$$

Result(type 7, 69 leaves):

$$\sum_{R=\text{RootOf}(Z^6 c + Z^3 b + a)} \frac{(cd - R^4 + (-ae + bd) - R) \ln(x - R)}{2 - R^5 c + -R^2 b} - \frac{d}{3 a} - \frac{d}{x a}$$

Problem 9: Result is not expressed in closed-form.

$$\int \frac{ex^3 + d}{x^3 (cx^6 + bx^3 + a)} dx$$

Optimal (type 3, 517 leaves, 14 steps):

$$\begin{aligned} & -\frac{d}{2ax^2} - \frac{c^{2/3} \ln\left(2^{1/3} c^{1/3} x + (b - \sqrt{-4ac + b^2})^{1/3}\right) \left(d + \frac{-2ae + bd}{\sqrt{-4ac + b^2}}\right) 2^{2/3}}{6a (b - \sqrt{-4ac + b^2})^{2/3}} \\ & + \frac{c^{2/3} \ln\left(2^{2/3} c^{2/3} x^2 - 2^{1/3} c^{1/3} x (b - \sqrt{-4ac + b^2})^{1/3} + (b - \sqrt{-4ac + b^2})^{2/3}\right) \left(d + \frac{-2ae + bd}{\sqrt{-4ac + b^2}}\right) 2^{2/3}}{12a (b - \sqrt{-4ac + b^2})^{2/3}} \\ & + \frac{c^{2/3} \arctan\left(\frac{\left(1 - \frac{2 \cdot 2^{1/3} c^{1/3} x}{(b - \sqrt{-4ac + b^2})^{1/3}}\right) \sqrt{3}}{3}\right) \left(d + \frac{-2ae + bd}{\sqrt{-4ac + b^2}}\right) 2^{2/3} \sqrt{3}}{6a (b - \sqrt{-4ac + b^2})^{2/3}} \\ & - \frac{c^{2/3} \ln\left(2^{1/3} c^{1/3} x + (b + \sqrt{-4ac + b^2})^{1/3}\right) \left(d + \frac{2ae - bd}{\sqrt{-4ac + b^2}}\right) 2^{2/3}}{6a (b + \sqrt{-4ac + b^2})^{2/3}} \\ & + \frac{c^{2/3} \ln\left(2^{2/3} c^{2/3} x^2 - 2^{1/3} c^{1/3} x (b + \sqrt{-4ac + b^2})^{1/3} + (b + \sqrt{-4ac + b^2})^{2/3}\right) \left(d + \frac{2ae - bd}{\sqrt{-4ac + b^2}}\right) 2^{2/3}}{12a (b + \sqrt{-4ac + b^2})^{2/3}} \\ & + \frac{c^{2/3} \arctan\left(\frac{\left(1 - \frac{2 \cdot 2^{1/3} c^{1/3} x}{(b + \sqrt{-4ac + b^2})^{1/3}}\right) \sqrt{3}}{3}\right) \left(d + \frac{2ae - bd}{\sqrt{-4ac + b^2}}\right) 2^{2/3} \sqrt{3}}{6a (b + \sqrt{-4ac + b^2})^{2/3}} \end{aligned}$$

Result (type 7, 67 leaves):

$$-\frac{d}{2ax^2} + \frac{\sum_{R=\text{RootOf}(Z^6 c + Z^3 b + a)} \frac{(-R^3 cd + ae - bd) \ln(x - R)}{2R^5 c + R^2 b}}{3a}$$

Problem 11: Result is not expressed in closed-form.

$$\int \frac{-x^3 + 1}{x^6 - x^3 + 1} dx$$

Optimal (type 3, 289 leaves, 13 steps):

$$\begin{aligned} & -\frac{\arctan\left(\frac{\left(1 + \frac{2 \cdot 2^{1/3} x}{(1 - I\sqrt{3})^{1/3}}\right) \sqrt{3}}{3}\right) (I - \sqrt{3}) 2^{2/3}}{6 (1 - I\sqrt{3})^{2/3}} - \frac{\ln(-2^{1/3} x + (1 - I\sqrt{3})^{1/3}) (3 - I\sqrt{3}) 2^{2/3}}{18 (1 - I\sqrt{3})^{2/3}} \\ & + \frac{\ln(2^{2/3} x^2 + 2^{1/3} (1 - I\sqrt{3})^{1/3} x + (1 - I\sqrt{3})^{2/3}) (3 - I\sqrt{3}) 2^{2/3}}{36 (1 - I\sqrt{3})^{2/3}} - \frac{\ln(-2^{1/3} x + (1 + I\sqrt{3})^{1/3}) (3 + I\sqrt{3}) 2^{2/3}}{18 (1 + I\sqrt{3})^{2/3}} \\ & + \frac{\ln(2^{2/3} x^2 + 2^{1/3} x (1 + I\sqrt{3})^{1/3} + (1 + I\sqrt{3})^{2/3}) (3 + I\sqrt{3}) 2^{2/3}}{36 (1 + I\sqrt{3})^{2/3}} + \frac{\arctan\left(\frac{\left(1 + \frac{2 \cdot 2^{1/3} x}{(1 + I\sqrt{3})^{1/3}}\right) \sqrt{3}}{3}\right) (I + \sqrt{3}) 2^{2/3}}{6 (1 + I\sqrt{3})^{2/3}} \end{aligned}$$

Result (type 7, 43 leaves):

$$\left(\frac{\sum_{R=\text{RootOf}(Z^6 - Z^3 + 1)} \frac{(-R^3 + 1) \ln(x - R)}{2R^5 - R^2}}{3} \right)$$

Problem 12: Result more than twice size of optimal antiderivative.

$$\int (ex^3 + d)^{5/2} (cx^6 + bx^3 + a) dx$$

Optimal (type 4, 321 leaves, 6 steps):

$$\begin{aligned} & \frac{30d(667ae^2 - 58bde + 16cd^2)x(ex^3 + d)^{3/2}}{124729e^2} + \frac{2(667ae^2 - 58bde + 16cd^2)x(ex^3 + d)^{5/2}}{11339e^2} - \frac{2(-29be + 8cd)x(ex^3 + d)^{7/2}}{667e^2} \\ & + \frac{2cx^4(ex^3 + d)^{7/2}}{29e} + \frac{54d^2(667ae^2 - 58bde + 16cd^2)x\sqrt{ex^3 + d}}{124729e^2} \end{aligned}$$

$$+ \frac{1}{124729 e^{7/3} \sqrt{ex^3+d} \sqrt{\frac{d^{1/3}(d^{1/3}+e^{1/3}x)}{(e^{1/3}x+d^{1/3}(1+\sqrt{3}))^2}}} \left(54 \cdot 3^{3/4} d^3 (667 a e^2 - 58 b d e + 16 c d^2) (d^{1/3} + e^{1/3} x) \operatorname{EllipticF} \left(\frac{e^{1/3} x + d^{1/3} (1 - \sqrt{3})}{e^{1/3} x + d^{1/3} (1 + \sqrt{3})}, I\sqrt{3} + 2I \right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} \right) \sqrt{\frac{d^{2/3} - d^{1/3} e^{1/3} x + e^{2/3} x^2}{(e^{1/3} x + d^{1/3} (1 + \sqrt{3}))^2}} \right)$$

Result (type 4, 1069 leaves):

$$a \left(\frac{2 e^2 x^7 \sqrt{ex^3+d}}{17} + \frac{74 d e x^4 \sqrt{ex^3+d}}{187} + \frac{106 d^2 x \sqrt{ex^3+d}}{187} - \frac{1}{187 e \sqrt{ex^3+d}} \left(54 I d^3 \sqrt{3} (-d e^2)^{1/3} \right. \right. \\ \left. \left. 3 \sqrt{\frac{I \left(x + \frac{(-d e^2)^{1/3}}{2 e} - \frac{I \sqrt{3} (-d e^2)^{1/3}}{2 e} \right) \sqrt{3} e}{(-d e^2)^{1/3}}} \sqrt{\frac{x - \frac{(-d e^2)^{1/3}}{e}}{-\frac{3 (-d e^2)^{1/3}}{2 e} + \frac{I \sqrt{3} (-d e^2)^{1/3}}{2 e}}} \right. \right. \\ \left. \left. \sqrt{\frac{-I \left(x + \frac{(-d e^2)^{1/3}}{2 e} + \frac{I \sqrt{3} (-d e^2)^{1/3}}{2 e} \right) \sqrt{3} e}{(-d e^2)^{1/3}}} \operatorname{EllipticF} \left(\frac{\sqrt{3} \sqrt{\frac{I \left(x + \frac{(-d e^2)^{1/3}}{2 e} - \frac{I \sqrt{3} (-d e^2)^{1/3}}{2 e} \right) \sqrt{3} e}{(-d e^2)^{1/3}}}}{3} \right), \right. \right. \\ \left. \left. \sqrt{\frac{I \sqrt{3} (-d e^2)^{1/3}}{e \left(-\frac{3 (-d e^2)^{1/3}}{2 e} + \frac{I \sqrt{3} (-d e^2)^{1/3}}{2 e} \right)}} \right) \right) + b \left(\frac{2 e^2 x^{10} \sqrt{ex^3+d}}{23} + \frac{98 d e x^7 \sqrt{ex^3+d}}{391} + \frac{974 d^2 x^4 \sqrt{ex^3+d}}{4301} + \frac{162 d^3 x \sqrt{ex^3+d}}{4301 e} \right. \\ \left. + \frac{1}{4301 e^2 \sqrt{ex^3+d}} \left(108 I d^4 \sqrt{3} (-d e^2)^{1/3} \right) \right)$$

$$\begin{aligned}
& \sqrt[3]{\frac{\operatorname{I}\left(x + \frac{(-de^2)^{1/3}}{2e} - \frac{\operatorname{I}\sqrt{3}(-de^2)^{1/3}}{2e}\right)\sqrt{3}e}{(-de^2)^{1/3}}} \sqrt{\frac{x - \frac{(-de^2)^{1/3}}{e}}{-\frac{3(-de^2)^{1/3}}{2e} + \frac{\operatorname{I}\sqrt{3}(-de^2)^{1/3}}{2e}}} \\
& \sqrt{\frac{-\operatorname{I}\left(x + \frac{(-de^2)^{1/3}}{2e} + \frac{\operatorname{I}\sqrt{3}(-de^2)^{1/3}}{2e}\right)\sqrt{3}e}{(-de^2)^{1/3}}} \operatorname{EllipticF}\left[\frac{\sqrt{3}\sqrt{\frac{\operatorname{I}\left(x + \frac{(-de^2)^{1/3}}{2e} - \frac{\operatorname{I}\sqrt{3}(-de^2)^{1/3}}{2e}\right)\sqrt{3}e}{(-de^2)^{1/3}}}}{3}\right], \\
& \left. \sqrt{\frac{\operatorname{I}\sqrt{3}(-de^2)^{1/3}}{e\left(-\frac{3(-de^2)^{1/3}}{2e} + \frac{\operatorname{I}\sqrt{3}(-de^2)^{1/3}}{2e}\right)}}\right) + c \left(\frac{2e^2x^{13}\sqrt{ex^3+d}}{29} + \frac{122dex^{10}\sqrt{ex^3+d}}{667} + \frac{1562d^2x^7\sqrt{ex^3+d}}{11339} + \frac{810d^3x^4\sqrt{ex^3+d}}{124729e} \right. \\
& \left. - \frac{1296d^4x\sqrt{ex^3+d}}{124729e^2} - \frac{1}{124729e^3\sqrt{ex^3+d}} \right) 864\operatorname{I}d^5\sqrt{3}(-de^2)^{1/3} \\
& \sqrt[3]{\frac{\operatorname{I}\left(x + \frac{(-de^2)^{1/3}}{2e} - \frac{\operatorname{I}\sqrt{3}(-de^2)^{1/3}}{2e}\right)\sqrt{3}e}{(-de^2)^{1/3}}} \sqrt{\frac{x - \frac{(-de^2)^{1/3}}{e}}{-\frac{3(-de^2)^{1/3}}{2e} + \frac{\operatorname{I}\sqrt{3}(-de^2)^{1/3}}{2e}}} \\
& \sqrt{\frac{-\operatorname{I}\left(x + \frac{(-de^2)^{1/3}}{2e} + \frac{\operatorname{I}\sqrt{3}(-de^2)^{1/3}}{2e}\right)\sqrt{3}e}{(-de^2)^{1/3}}} \operatorname{EllipticF}\left[\frac{\sqrt{3}\sqrt{\frac{\operatorname{I}\left(x + \frac{(-de^2)^{1/3}}{2e} - \frac{\operatorname{I}\sqrt{3}(-de^2)^{1/3}}{2e}\right)\sqrt{3}e}{(-de^2)^{1/3}}}}{3}\right],
\end{aligned}$$

$$\sqrt{\frac{\frac{I\sqrt{3}(-de^2)^{1/3}}{2e}}{e\left(-\frac{3(-de^2)^{1/3}}{2e} + \frac{I\sqrt{3}(-de^2)^{1/3}}{2e}\right)}}}$$

Problem 13: Result more than twice size of optimal antiderivative.

$$\int \frac{cx^6 + bx^3 + a}{(ex^3 + d)^{3/2}} dx$$

Optimal (type 4, 226 leaves, 3 steps):

$$\frac{2(ae^2 - bde + cd^2)x}{3de^2\sqrt{ex^3 + d}} + \frac{2cx\sqrt{ex^3 + d}}{5e^2} - \frac{1}{45de^{7/3}\sqrt{ex^3 + d}} \sqrt{\frac{d^{1/3}(d^{1/3} + e^{1/3}x)}{(e^{1/3}x + d^{1/3}(1 + \sqrt{3}))^2}} \left(2(16cd^2 - 5e(ae + 2bd))(d^{1/3} + e^{1/3}x) \operatorname{EllipticF}\left(\frac{e^{1/3}x + d^{1/3}(1 - \sqrt{3})}{e^{1/3}x + d^{1/3}(1 + \sqrt{3})}\right), \right. \\ \left. I\sqrt{3} + 2I\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} \right) \sqrt{\frac{d^{2/3} - d^{1/3}e^{1/3}x + e^{2/3}x^2}{(e^{1/3}x + d^{1/3}(1 + \sqrt{3}))^2}} 3^{3/4}$$

Result (type 4, 933 leaves):

$$a \left(\frac{2x}{3d\sqrt{\left(x^3 + \frac{d}{e}\right)e}} - \frac{1}{9de\sqrt{ex^3 + d}} \right) \left(2I\sqrt{3}(-de^2)^{1/3} \right. \\ \left. \sqrt[3]{\frac{I\left(x + \frac{(-de^2)^{1/3}}{2e} - \frac{I\sqrt{3}(-de^2)^{1/3}}{2e}\right)\sqrt{3}e}{(-de^2)^{1/3}}} \sqrt{\frac{x - \frac{(-de^2)^{1/3}}{e}}{-\frac{3(-de^2)^{1/3}}{2e} + \frac{I\sqrt{3}(-de^2)^{1/3}}{2e}}}} \right)$$

$$\sqrt{\frac{-I\left(x + \frac{(-de^2)^{1/3}}{2e} + \frac{I\sqrt{3}(-de^2)^{1/3}}{2e}\right)\sqrt{3}e}{(-de^2)^{1/3}}} \text{EllipticF} \left(\frac{\sqrt{3} \sqrt{\frac{I\left(x + \frac{(-de^2)^{1/3}}{2e} - \frac{I\sqrt{3}(-de^2)^{1/3}}{2e}\right)\sqrt{3}e}{(-de^2)^{1/3}}}}{3}, \right.$$

$$\left. \left. \sqrt{\frac{I\sqrt{3}(-de^2)^{1/3}}{e\left(-\frac{3(-de^2)^{1/3}}{2e} + \frac{I\sqrt{3}(-de^2)^{1/3}}{2e}\right)}} \right) \right) + b \left(-\frac{2x}{3e\sqrt{\left(x^3 + \frac{d}{e}\right)e}} - \frac{1}{9e^2\sqrt{ex^3+d}} \right) 4I\sqrt{3}(-de^2)^{1/3}$$

$$3 \sqrt{\frac{I\left(x + \frac{(-de^2)^{1/3}}{2e} - \frac{I\sqrt{3}(-de^2)^{1/3}}{2e}\right)\sqrt{3}e}{(-de^2)^{1/3}}} \sqrt{\frac{x - \frac{(-de^2)^{1/3}}{e}}{-\frac{3(-de^2)^{1/3}}{2e} + \frac{I\sqrt{3}(-de^2)^{1/3}}{2e}}}$$

$$\sqrt{\frac{-I\left(x + \frac{(-de^2)^{1/3}}{2e} + \frac{I\sqrt{3}(-de^2)^{1/3}}{2e}\right)\sqrt{3}e}{(-de^2)^{1/3}}} \text{EllipticF} \left(\frac{\sqrt{3} \sqrt{\frac{I\left(x + \frac{(-de^2)^{1/3}}{2e} - \frac{I\sqrt{3}(-de^2)^{1/3}}{2e}\right)\sqrt{3}e}{(-de^2)^{1/3}}}}{3}, \right.$$

$$\left. \left. \sqrt{\frac{I\sqrt{3}(-de^2)^{1/3}}{e\left(-\frac{3(-de^2)^{1/3}}{2e} + \frac{I\sqrt{3}(-de^2)^{1/3}}{2e}\right)}} \right) \right) + c \left(\frac{2dx}{3e^2\sqrt{\left(x^3 + \frac{d}{e}\right)e}} + \frac{2x\sqrt{ex^3+d}}{5e^2} + \frac{1}{45e^3\sqrt{ex^3+d}} \right) 32Id\sqrt{3}(-de^2)^{1/3}$$

$$3 \sqrt{\frac{I\left(x + \frac{(-de^2)^{1/3}}{2e} - \frac{I\sqrt{3}(-de^2)^{1/3}}{2e}\right)\sqrt{3}e}{(-de^2)^{1/3}}} \sqrt{\frac{x - \frac{(-de^2)^{1/3}}{e}}{-\frac{3(-de^2)^{1/3}}{2e} + \frac{I\sqrt{3}(-de^2)^{1/3}}{2e}}}$$

$$\sqrt{\frac{-1 \left(x + \frac{(-de^2)^{1/3}}{2e} + \frac{I\sqrt{3}(-de^2)^{1/3}}{2e} \right) \sqrt{3} e}{(-de^2)^{1/3}}} \operatorname{EllipticF} \left(\frac{\sqrt{3} \sqrt{\frac{I \left(x + \frac{(-de^2)^{1/3}}{2e} - \frac{I\sqrt{3}(-de^2)^{1/3}}{2e} \right) \sqrt{3} e}{(-de^2)^{1/3}}}}{3} \right),$$

$$\left. \left. \left. \left. \left. \sqrt{\frac{I\sqrt{3}(-de^2)^{1/3}}{e \left(-\frac{3(-de^2)^{1/3}}{2e} + \frac{I\sqrt{3}(-de^2)^{1/3}}{2e} \right)}} \right) \right) \right) \right) \right)$$

Problem 14: Result more than twice size of optimal antiderivative.

$$\int \frac{cx^6 + bx^3 + a}{(ex^3 + d)^{7/2}} dx$$

Optimal (type 4, 282 leaves, 4 steps):

$$\frac{2(ae^2 - bde + cd^2)x}{15d^2e^2(ex^3 + d)^{5/2}} - \frac{2(-13ae^2 - 2bde + 17cd^2)x}{135d^2e^2(ex^3 + d)^3/2} + \frac{2(91ae^2 + 14bde + 16cd^2)x}{405d^3e^2\sqrt{ex^3 + d}}$$

$$+ \frac{1}{1215d^3e^{7/3}\sqrt{ex^3 + d}} \sqrt{\frac{d^{1/3}(d^{1/3} + e^{1/3}x)}{(e^{1/3}x + d^{1/3}(1 + \sqrt{3}))^2}} \left(2(91ae^2 + 14bde + 16cd^2)(d^{1/3} + e^{1/3}x) \operatorname{EllipticF} \left(\frac{e^{1/3}x + d^{1/3}(1 - \sqrt{3})}{e^{1/3}x + d^{1/3}(1 + \sqrt{3})}, I\sqrt{3} + 2I \right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} \right) \sqrt{\frac{d^{2/3} - d^{1/3}e^{1/3}x + e^{2/3}x^2}{(e^{1/3}x + d^{1/3}(1 + \sqrt{3}))^2}} \right)^{3/4}$$

Result (type 4, 1094 leaves):

$$a \left(\frac{2x\sqrt{ex^3 + d}}{15d^2e^3 \left(x^3 + \frac{d}{e} \right)^3} + \frac{26x\sqrt{ex^3 + d}}{135d^2e^2 \left(x^3 + \frac{d}{e} \right)^2} + \frac{182x}{405d^3 \sqrt{\left(x^3 + \frac{d}{e} \right) e}} - \frac{1}{1215d^3e\sqrt{ex^3 + d}} \right) 182I\sqrt{3}(-de^2)^{1/3}$$

$$\begin{aligned}
& \sqrt[3]{\frac{\operatorname{I}\left(x + \frac{(-de^2)^{1/3}}{2e} - \frac{\operatorname{I}\sqrt{3}(-de^2)^{1/3}}{2e}\right)\sqrt{3}e}{(-de^2)^{1/3}}} \sqrt{\frac{x - \frac{(-de^2)^{1/3}}{e}}{-\frac{3(-de^2)^{1/3}}{2e} + \frac{\operatorname{I}\sqrt{3}(-de^2)^{1/3}}{2e}}} \\
& \sqrt{\frac{-\operatorname{I}\left(x + \frac{(-de^2)^{1/3}}{2e} + \frac{\operatorname{I}\sqrt{3}(-de^2)^{1/3}}{2e}\right)\sqrt{3}e}{(-de^2)^{1/3}}} \operatorname{EllipticF}\left[\frac{\sqrt[3]{\frac{\operatorname{I}\left(x + \frac{(-de^2)^{1/3}}{2e} - \frac{\operatorname{I}\sqrt{3}(-de^2)^{1/3}}{2e}\right)\sqrt{3}e}{(-de^2)^{1/3}}}}{3},\right. \\
& \left.\sqrt{\frac{\operatorname{I}\sqrt{3}(-de^2)^{1/3}}{e\left(-\frac{3(-de^2)^{1/3}}{2e} + \frac{\operatorname{I}\sqrt{3}(-de^2)^{1/3}}{2e}\right)}}}\right] + b\left[-\frac{2x\sqrt{ex^3+d}}{15e^4\left(x^3+\frac{d}{e}\right)^3} + \frac{4x\sqrt{ex^3+d}}{135de^3\left(x^3+\frac{d}{e}\right)^2} + \frac{28x}{405ed^2\sqrt{\left(x^3+\frac{d}{e}\right)e}}\right. \\
& \left. - \frac{1}{1215e^2d^2\sqrt{ex^3+d}}\right] 28\operatorname{I}\sqrt{3}(-de^2)^{1/3} \\
& \sqrt[3]{\frac{\operatorname{I}\left(x + \frac{(-de^2)^{1/3}}{2e} - \frac{\operatorname{I}\sqrt{3}(-de^2)^{1/3}}{2e}\right)\sqrt{3}e}{(-de^2)^{1/3}}} \sqrt{\frac{x - \frac{(-de^2)^{1/3}}{e}}{-\frac{3(-de^2)^{1/3}}{2e} + \frac{\operatorname{I}\sqrt{3}(-de^2)^{1/3}}{2e}}} \\
& \sqrt{\frac{-\operatorname{I}\left(x + \frac{(-de^2)^{1/3}}{2e} + \frac{\operatorname{I}\sqrt{3}(-de^2)^{1/3}}{2e}\right)\sqrt{3}e}{(-de^2)^{1/3}}} \operatorname{EllipticF}\left[\frac{\sqrt[3]{\frac{\operatorname{I}\left(x + \frac{(-de^2)^{1/3}}{2e} - \frac{\operatorname{I}\sqrt{3}(-de^2)^{1/3}}{2e}\right)\sqrt{3}e}{(-de^2)^{1/3}}}}{3},\right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \left. \sqrt{\frac{I\sqrt{3}(-de^2)^{1/3}}{e\left(-\frac{3(-de^2)^{1/3}}{2e} + \frac{I\sqrt{3}(-de^2)^{1/3}}{2e}\right)}}}\right)\right)\right) + c \left(\frac{2dx\sqrt{ex^3+d}}{15e^5\left(x^3+\frac{d}{e}\right)^3} - \frac{34x\sqrt{ex^3+d}}{135e^4\left(x^3+\frac{d}{e}\right)^2} + \frac{32x}{405e^2d\sqrt{\left(x^3+\frac{d}{e}\right)e}} \right) \\
& - \frac{1}{1215e^3d\sqrt{ex^3+d}} \left(32I\sqrt{3}(-de^2)^{1/3} \right. \\
& \left. \sqrt[3]{\frac{I\left(x + \frac{(-de^2)^{1/3}}{2e} - \frac{I\sqrt{3}(-de^2)^{1/3}}{2e}\right)\sqrt{3}e}{(-de^2)^{1/3}}} \right) \sqrt{\frac{x - \frac{(-de^2)^{1/3}}{e}}{-\frac{3(-de^2)^{1/3}}{2e} + \frac{I\sqrt{3}(-de^2)^{1/3}}{2e}}} \\
& \left. \sqrt{\frac{-I\left(x + \frac{(-de^2)^{1/3}}{2e} + \frac{I\sqrt{3}(-de^2)^{1/3}}{2e}\right)\sqrt{3}e}{(-de^2)^{1/3}}} \right) \text{EllipticF} \left(\frac{\sqrt{3} \sqrt{\frac{I\left(x + \frac{(-de^2)^{1/3}}{2e} - \frac{I\sqrt{3}(-de^2)^{1/3}}{2e}\right)\sqrt{3}e}{(-de^2)^{1/3}}}}}{3} \right), \\
& \left. \left. \left. \sqrt{\frac{I\sqrt{3}(-de^2)^{1/3}}{e\left(-\frac{3(-de^2)^{1/3}}{2e} + \frac{I\sqrt{3}(-de^2)^{1/3}}{2e}\right)}}}\right)\right)\right)
\end{aligned}$$

Problem 15: Result is not expressed in closed-form.

$$\int \frac{x^2(ex^4+d)}{cx^8+bx^4+a} dx$$

Optimal (type 3, 291 leaves, 7 steps):

$$\frac{\arctan\left(\frac{2^{1/4}c^{1/4}x}{(-b-\sqrt{-4ac+b^2})^{1/4}}\right)\left(e+\frac{be-2cd}{\sqrt{-4ac+b^2}}\right)^{2^{1/4}}}{4c^{3/4}(-b-\sqrt{-4ac+b^2})^{1/4}} - \frac{\operatorname{arctanh}\left(\frac{2^{1/4}c^{1/4}x}{(-b-\sqrt{-4ac+b^2})^{1/4}}\right)\left(e+\frac{be-2cd}{\sqrt{-4ac+b^2}}\right)^{2^{1/4}}}{4c^{3/4}(-b-\sqrt{-4ac+b^2})^{1/4}}$$

$$+ \frac{\arctan\left(\frac{2^{1/4} c^{1/4} x}{(-b + \sqrt{-4ac + b^2})^{1/4}}\right) \left(e + \frac{-be + 2cd}{\sqrt{-4ac + b^2}}\right)^{2^{1/4}}}{4c^{3/4} (-b + \sqrt{-4ac + b^2})^{1/4}} - \frac{\operatorname{arctanh}\left(\frac{2^{1/4} c^{1/4} x}{(-b + \sqrt{-4ac + b^2})^{1/4}}\right) \left(e + \frac{-be + 2cd}{\sqrt{-4ac + b^2}}\right)^{2^{1/4}}}{4c^{3/4} (-b + \sqrt{-4ac + b^2})^{1/4}}$$

Result(type 7, 50 leaves):

$$\frac{\left(\sum_{R=\text{RootOf}(Z^8 c + Z^4 b + a)} \frac{(_R^6 e + _R^2 d) \ln(x - _R)}{2 _R^7 c + _R^3 b}\right)}{4}$$

Problem 16: Result is not expressed in closed-form.

$$\int \frac{ex^4 + d}{cx^8 + bx^4 + a} dx$$

Optimal(type 3, 291 leaves, 7 steps):

$$\frac{\arctan\left(\frac{2^{1/4} c^{1/4} x}{(-b - \sqrt{-4ac + b^2})^{1/4}}\right) \left(e + \frac{be - 2cd}{\sqrt{-4ac + b^2}}\right)^{2^{3/4}}}{4c^{1/4} (-b - \sqrt{-4ac + b^2})^{3/4}} - \frac{\operatorname{arctanh}\left(\frac{2^{1/4} c^{1/4} x}{(-b - \sqrt{-4ac + b^2})^{1/4}}\right) \left(e + \frac{be - 2cd}{\sqrt{-4ac + b^2}}\right)^{2^{3/4}}}{4c^{1/4} (-b - \sqrt{-4ac + b^2})^{3/4}} - \frac{\arctan\left(\frac{2^{1/4} c^{1/4} x}{(-b + \sqrt{-4ac + b^2})^{1/4}}\right) \left(e + \frac{-be + 2cd}{\sqrt{-4ac + b^2}}\right)^{2^{3/4}}}{4c^{1/4} (-b + \sqrt{-4ac + b^2})^{3/4}} - \frac{\operatorname{arctanh}\left(\frac{2^{1/4} c^{1/4} x}{(-b + \sqrt{-4ac + b^2})^{1/4}}\right) \left(e + \frac{-be + 2cd}{\sqrt{-4ac + b^2}}\right)^{2^{3/4}}}{4c^{1/4} (-b + \sqrt{-4ac + b^2})^{3/4}}$$

Result(type 7, 46 leaves):

$$\frac{\left(\sum_{R=\text{RootOf}(Z^8 c + Z^4 b + a)} \frac{(_R^4 e + d) \ln(x - _R)}{2 _R^7 c + _R^3 b}\right)}{4}$$

Problem 17: Result is not expressed in closed-form.

$$\int \frac{ex^4 + d}{x^2 (cx^8 + bx^4 + a)} dx$$

Optimal(type 3, 312 leaves, 8 steps):

$$-\frac{d}{xa} - \frac{c^{1/4} \arctan\left(\frac{2^{1/4} c^{1/4} x}{(-b - \sqrt{-4ac + b^2})^{1/4}}\right) \left(d + \frac{2ae - bd}{\sqrt{-4ac + b^2}}\right)^{2^{1/4}}}{4a (-b - \sqrt{-4ac + b^2})^{1/4}} + \frac{c^{1/4} \operatorname{arctanh}\left(\frac{2^{1/4} c^{1/4} x}{(-b - \sqrt{-4ac + b^2})^{1/4}}\right) \left(d + \frac{2ae - bd}{\sqrt{-4ac + b^2}}\right)^{2^{1/4}}}{4a (-b - \sqrt{-4ac + b^2})^{1/4}}$$

$$-\frac{c^{1/4} \arctan\left(\frac{2^{1/4} c^{1/4} x}{(-b + \sqrt{-4ac + b^2})^{1/4}}\right) \left(d + \frac{-2ae + bd}{\sqrt{-4ac + b^2}}\right)^{2^{1/4}}}{4a(-b + \sqrt{-4ac + b^2})^{1/4}} + \frac{c^{1/4} \operatorname{arctanh}\left(\frac{2^{1/4} c^{1/4} x}{(-b + \sqrt{-4ac + b^2})^{1/4}}\right) \left(d + \frac{-2ae + bd}{\sqrt{-4ac + b^2}}\right)^{2^{1/4}}}{4a(-b + \sqrt{-4ac + b^2})^{1/4}}$$

Result(type 7, 71 leaves):

$$-\frac{\sum_{R=\text{RootOf}(Z^8 c + Z^4 b + a)} \frac{(cd R^6 + (-ae + bd) R^2) \ln(x - R)}{2 R^7 c + R^3 b}}{4a} - \frac{d}{xa}$$

Problem 18: Result is not expressed in closed-form.

$$\int \frac{-x^4 + 1}{x^8 - x^4 + 1} dx$$

Optimal(type 3, 307 leaves, 19 steps):

$$\frac{\ln\left(1 + x^2 - x\left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right)\right) \left(\frac{\sqrt{2}}{2} - \frac{\sqrt{6}}{6}\right)}{8} - \frac{\ln\left(1 + x^2 + x\left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right)\right) \left(\frac{\sqrt{2}}{2} - \frac{\sqrt{6}}{6}\right)}{8} - \frac{\arctan\left(\frac{-2x + \frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}}{\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}}\right)}{4\left(\frac{3\sqrt{2}}{2} - \frac{\sqrt{6}}{2}\right)}$$

$$+ \frac{\arctan\left(\frac{2x + \frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}}{\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}}\right)}{4\left(\frac{3\sqrt{2}}{2} - \frac{\sqrt{6}}{2}\right)} - \frac{\ln\left(1 + x^2 - x\left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right)\right) \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{6}}{6}\right)}{8} + \frac{\ln\left(1 + x^2 + x\left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right)\right) \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{6}}{6}\right)}{8}$$

$$+ \frac{\arctan\left(\frac{-2x + \frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}}{\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}}\right)}{4\left(\frac{3\sqrt{2}}{2} + \frac{\sqrt{6}}{2}\right)} - \frac{\arctan\left(\frac{2x + \frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}}{\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}}\right)}{4\left(\frac{3\sqrt{2}}{2} + \frac{\sqrt{6}}{2}\right)}$$

Result(type 7, 43 leaves):

$$\frac{\left(\sum_{R=\text{RootOf}(Z^8 - Z^4 + 1)} \frac{(-R^4 + 1) \ln(x - R)}{2 R^7 - R^3}\right)}{4}$$

Problem 23: Result more than twice size of optimal antiderivative.

$$\int \frac{x^3}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right) (ex + d)^2} dx$$

Optimal (type 3, 335 leaves, 7 steps):

$$\begin{aligned} & -\frac{(2ad+be)x}{a^2e^3} + \frac{x^2}{2ae^2} + \frac{d^5}{e^4(a^2d^2 - e(bd - ce))(ex+d)} + \frac{d^4(3ad^2 - e(4bd - 5ce)) \ln(ex+d)}{e^4(a^2d^2 - e(bd - ce))^2} \\ & + \frac{(b^4d^2 - 2b^3cde + 4abc^2de + ac^2(a^2d^2 - ce^2) - b^2c(3ad^2 - ce^2)) \ln(ax^2 + bx + c)}{2a^3(a^2d^2 - e(bd - ce))^2} \\ & + \frac{(b^5d^2 - 2b^4cde + 8ab^2c^2de - 4a^2c^3de + abc^2(5ad^2 - 3ce^2) - b^3c(5ad^2 - ce^2)) \operatorname{arctanh}\left(\frac{2xa+b}{\sqrt{-4ac+b^2}}\right)}{a^3(a^2d^2 - e(bd - ce))^2 \sqrt{-4ac+b^2}} \end{aligned}$$

Result (type 3, 942 leaves):

$$\begin{aligned} & \frac{x^2}{2ae^2} - \frac{2dx}{ae^3} - \frac{bx}{a^2e^2} + \frac{\ln(ax^2 + bx + c) d^2 c^2}{2(a^2d^2 - bde + ce^2)^2 a} - \frac{3 \ln(ax^2 + bx + c) b^2 c d^2}{2(a^2d^2 - bde + ce^2)^2 a^2} + \frac{2 \ln(ax^2 + bx + c) b c^2 de}{(a^2d^2 - bde + ce^2)^2 a^2} - \frac{\ln(ax^2 + bx + c) c^3 e^2}{2(a^2d^2 - bde + ce^2)^2 a^2} \\ & + \frac{\ln(ax^2 + bx + c) b^4 d^2}{2(a^2d^2 - bde + ce^2)^2 a^3} - \frac{\ln(ax^2 + bx + c) b^3 cde}{(a^2d^2 - bde + ce^2)^2 a^3} + \frac{\ln(ax^2 + bx + c) b^2 c^2 e^2}{2(a^2d^2 - bde + ce^2)^2 a^3} - \frac{5 \operatorname{arctan}\left(\frac{2xa+b}{\sqrt{4ac-b^2}}\right) b c^2 d^2}{(a^2d^2 - bde + ce^2)^2 a \sqrt{4ac-b^2}} \\ & + \frac{4 \operatorname{arctan}\left(\frac{2xa+b}{\sqrt{4ac-b^2}}\right) c^3 de}{(a^2d^2 - bde + ce^2)^2 a \sqrt{4ac-b^2}} + \frac{5 \operatorname{arctan}\left(\frac{2xa+b}{\sqrt{4ac-b^2}}\right) b^3 c d^2}{(a^2d^2 - bde + ce^2)^2 a^2 \sqrt{4ac-b^2}} - \frac{8 \operatorname{arctan}\left(\frac{2xa+b}{\sqrt{4ac-b^2}}\right) b^2 c^2 de}{(a^2d^2 - bde + ce^2)^2 a^2 \sqrt{4ac-b^2}} \\ & + \frac{3 \operatorname{arctan}\left(\frac{2xa+b}{\sqrt{4ac-b^2}}\right) b c^3 e^2}{(a^2d^2 - bde + ce^2)^2 a^2 \sqrt{4ac-b^2}} - \frac{\operatorname{arctan}\left(\frac{2xa+b}{\sqrt{4ac-b^2}}\right) b^5 d^2}{(a^2d^2 - bde + ce^2)^2 a^3 \sqrt{4ac-b^2}} + \frac{2 \operatorname{arctan}\left(\frac{2xa+b}{\sqrt{4ac-b^2}}\right) b^4 cde}{(a^2d^2 - bde + ce^2)^2 a^3 \sqrt{4ac-b^2}} \\ & - \frac{\operatorname{arctan}\left(\frac{2xa+b}{\sqrt{4ac-b^2}}\right) b^3 c^2 e^2}{(a^2d^2 - bde + ce^2)^2 a^3 \sqrt{4ac-b^2}} + \frac{3d^6 \ln(ex+d) a}{e^4(a^2d^2 - bde + ce^2)^2} - \frac{4d^5 \ln(ex+d) b}{e^3(a^2d^2 - bde + ce^2)^2} + \frac{5d^4 \ln(ex+d) c}{e^2(a^2d^2 - bde + ce^2)^2} \\ & + \frac{d^5}{e^4(a^2d^2 - bde + ce^2)(ex+d)} \end{aligned}$$

Problem 24: Result more than twice size of optimal antiderivative.

$$\int x^3 \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{ex+d} dx$$

Optimal(type 4, 702 leaves, 10 steps):

$$\begin{aligned}
 & - \frac{4(8a^2d^2 + 3b^2e^2 + ae(4bd - 7ce))x(ex+d)^3 \sqrt{a + \frac{c}{x^2} + \frac{b}{x}}}{315a^2e^3} + \frac{2(ad+be)x(ex+d)^5 \sqrt{a + \frac{c}{x^2} + \frac{b}{x}}}{63ae^3} \\
 & + \frac{2(19a^3d^3 - 6a^2cde^2 + 8b^3e^3 + 3ab^2e^2(bd - 9ce))x \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{ex+d}}{315a^3e^3} + \frac{2x^4 \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{ex+d}}{9} \\
 & - \frac{1}{315a^4e^4(ax^2+bx+c) \sqrt{\frac{a(ex+d)}{2ad-e(b+\sqrt{-4ac+b^2})}}} \left(2(8a^4d^4 + 8b^4e^4 - a^3d^2e(4bd - 9ce) - 4ab^2e^3(bd + 9ce) - 3a^2e^2(b^2d^2 \right. \\
 & \left. - 5bcde - 7c^2e^2))x \text{EllipticE} \left(\frac{\sqrt{\frac{b+2xa+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}} \sqrt{2}}{2}, \right. \right. \\
 & \left. \left. \sqrt{-\frac{2e\sqrt{-4ac+b^2}}{2ad-e(b+\sqrt{-4ac+b^2})}} \sqrt{2} \sqrt{-4ac+b^2} \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{ex+d} \sqrt{-\frac{a(ax^2+bx+c)}{-4ac+b^2}} \right) \right) \\
 & + \frac{1}{315a^4e^4(ax^2+bx+c) \sqrt{ex+d}} \left(2(16a^3d^3 + 6a^2cde^2 - 8b^3e^3 - 3ab^2e^2(bd - 9ce))(ad^2 - e(bd \right. \\
 & \left. - ce))x \text{EllipticF} \left(\frac{\sqrt{\frac{b+2xa+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}} \sqrt{2}}{2}, \right. \right. \\
 & \left. \left. \sqrt{-\frac{2e\sqrt{-4ac+b^2}}{2ad-e(b+\sqrt{-4ac+b^2})}} \sqrt{2} \sqrt{-4ac+b^2} \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{-\frac{a(ax^2+bx+c)}{-4ac+b^2}} \sqrt{\frac{a(ex+d)}{2ad-e(b+\sqrt{-4ac+b^2})}} \right) \right)
 \end{aligned}$$

Result(type ?, 9181 leaves): Display of huge result suppressed!

Problem 25: Result more than twice size of optimal antiderivative.

$$\int x^2 \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{ex+d} \, dx$$

Optimal(type 4, 572 leaves, 8 steps):

$$\begin{aligned} & - \frac{2x(4a^2d^2 + 4b^2e^2 - ae(2bd - 5ce) - 3ae(ad - 4be)x) \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{ex+d}}{105a^2e^2} + \frac{2x(ax^2 + bx + c) \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{ex+d}}{7a} \\ & + \frac{1}{105a^3e^3(ax^2 + bx + c) \sqrt{\frac{a(ex+d)}{2ad - e(b + \sqrt{-4ac + b^2})}}} \left((8a^3d^3 + 8b^3e^3 - a^2de(5bd - 16ce) - abe^2(5bd \right. \\ & \left. + 29ce)) x \text{EllipticE} \left(\frac{\sqrt{\frac{b + 2xa + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}}{2}, \right. \right. \\ & \left. \left. \sqrt{-\frac{2e\sqrt{-4ac + b^2}}{2ad - e(b + \sqrt{-4ac + b^2})}} \right) \sqrt{2} \sqrt{-4ac + b^2} \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{ex+d} \sqrt{-\frac{a(ax^2 + bx + c)}{-4ac + b^2}} \right) \\ & - \frac{1}{105a^3e^3(ax^2 + bx + c) \sqrt{ex+d}} \left(2(8a^2d^2 - 4b^2e^2 - ae(bd - 10ce))(ad^2 - e(bd - ce)) x \text{EllipticF} \left(\frac{\sqrt{\frac{b + 2xa + \sqrt{-4ac + b^2}}{\sqrt{-4ac + b^2}}} \sqrt{2}}{2}, \right. \right. \\ & \left. \left. \sqrt{-\frac{2e\sqrt{-4ac + b^2}}{2ad - e(b + \sqrt{-4ac + b^2})}} \right) \sqrt{2} \sqrt{-4ac + b^2} \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{-\frac{a(ax^2 + bx + c)}{-4ac + b^2}} \sqrt{\frac{a(ex+d)}{2ad - e(b + \sqrt{-4ac + b^2})}} \right) \end{aligned}$$

Result(type ?, 6301 leaves): Display of huge result suppressed!

Problem 26: Unable to integrate problem.

$$\int \frac{(fx)^m (a + cx^{2n})^p}{(d + ex^n)^2} \, dx$$

Optimal(type 6, 290 leaves, 8 steps):

$$\begin{aligned}
& \frac{x (fx)^m (a + cx^{2n})^p \operatorname{AppellF1}\left(\frac{1+m}{2n}, 2, -p, 1 + \frac{1+m}{2n}, \frac{e^2 x^{2n}}{d^2}, -\frac{cx^{2n}}{a}\right)}{d^2 (1+m) \left(1 + \frac{cx^{2n}}{a}\right)^p} \\
& - \frac{2ex^{1+n} (fx)^m (a + cx^{2n})^p \operatorname{AppellF1}\left(\frac{1+m+n}{2n}, 2, -p, \frac{1+m+3n}{2n}, \frac{e^2 x^{2n}}{d^2}, -\frac{cx^{2n}}{a}\right)}{d^3 (1+m+n) \left(1 + \frac{cx^{2n}}{a}\right)^p} \\
& + \frac{e^2 x^{1+2n} (fx)^m (a + cx^{2n})^p \operatorname{AppellF1}\left(\frac{1+m+2n}{2n}, 2, -p, \frac{1+m+4n}{2n}, \frac{e^2 x^{2n}}{d^2}, -\frac{cx^{2n}}{a}\right)}{d^4 (1+m+2n) \left(1 + \frac{cx^{2n}}{a}\right)^p}
\end{aligned}$$

Result(type 8, 28 leaves):

$$\int \frac{(fx)^m (a + cx^{2n})^p}{(d + ex^n)^2} dx$$

Problem 27: Unable to integrate problem.

$$\int \frac{(fx)^m (a + cx^{2n})^p}{(d + ex^n)^3} dx$$

Optimal(type 6, 396 leaves, 10 steps):

$$\begin{aligned}
& \frac{x (fx)^m (a + cx^{2n})^p \operatorname{AppellF1}\left(\frac{1+m}{2n}, 3, -p, 1 + \frac{1+m}{2n}, \frac{e^2 x^{2n}}{d^2}, -\frac{cx^{2n}}{a}\right)}{d^3 (1+m) \left(1 + \frac{cx^{2n}}{a}\right)^p} \\
& - \frac{3ex^{1+n} (fx)^m (a + cx^{2n})^p \operatorname{AppellF1}\left(\frac{1+m+n}{2n}, 3, -p, \frac{1+m+3n}{2n}, \frac{e^2 x^{2n}}{d^2}, -\frac{cx^{2n}}{a}\right)}{d^4 (1+m+n) \left(1 + \frac{cx^{2n}}{a}\right)^p} \\
& + \frac{3e^2 x^{1+2n} (fx)^m (a + cx^{2n})^p \operatorname{AppellF1}\left(\frac{1+m+2n}{2n}, 3, -p, \frac{1+m+4n}{2n}, \frac{e^2 x^{2n}}{d^2}, -\frac{cx^{2n}}{a}\right)}{d^5 (1+m+2n) \left(1 + \frac{cx^{2n}}{a}\right)^p}
\end{aligned}$$

$$\frac{e^3 x^{1+3n} (fx)^m (a + cx^{2n})^p \text{AppellF1}\left(\frac{1+m+3n}{2n}, 3, -p, \frac{1+m+5n}{2n}, \frac{e^2 x^{2n}}{d^2}, -\frac{cx^{2n}}{a}\right)}{d^6 (1+m+3n) \left(1 + \frac{cx^{2n}}{a}\right)^p}$$

Result(type 8, 28 leaves):

$$\int \frac{(fx)^m (a + cx^{2n})^p}{(d + ex^n)^3} dx$$

Problem 28: Result more than twice size of optimal antiderivative.

$$\int x^{-1+n} (b + 2cx^n) (-a + bx^n + cx^{2n})^{13} dx$$

Optimal(type 3, 23 leaves, 2 steps):

$$\frac{(a - bx^n - cx^{2n})^{14}}{14n}$$

Result(type ?, 2045 leaves): Display of huge result suppressed!

Problem 29: Result more than twice size of optimal antiderivative.

$$\int (2cx + b) (cx^2 + bx)^{13} dx$$

Optimal(type 1, 13 leaves, 1 step):

$$\frac{(cx^2 + bx)^{14}}{14}$$

Result(type 1, 154 leaves):

$$\begin{aligned} & \frac{1}{14} c^{14} x^{28} + b c^{13} x^{27} + \frac{13}{2} b^2 c^{12} x^{26} + 26 b^3 c^{11} x^{25} + \frac{143}{2} b^4 c^{10} x^{24} + 143 b^5 c^9 x^{23} + \frac{429}{2} b^6 c^8 x^{22} + \frac{1716}{7} b^7 c^7 x^{21} + \frac{429}{2} b^8 c^6 x^{20} + 143 b^9 c^5 x^{19} \\ & + \frac{143}{2} b^{10} c^4 x^{18} + 26 b^{11} c^3 x^{17} + \frac{13}{2} b^{12} c^2 x^{16} + b^{13} c x^{15} + \frac{1}{14} b^{14} x^{14} \end{aligned}$$

Problem 30: Result more than twice size of optimal antiderivative.

$$\int x (2cx^2 + b) (cx^4 + bx^2)^{13} dx$$

Optimal(type 1, 14 leaves, 3 steps):

$$\frac{x^{28} (cx^2 + b)^{14}}{28}$$

Result(type 1, 156 leaves):

$$\frac{1}{28} c^{14} x^{56} + \frac{1}{2} b c^{13} x^{54} + \frac{13}{4} b^2 c^{12} x^{52} + 13 b^3 c^{11} x^{50} + \frac{143}{4} b^4 c^{10} x^{48} + \frac{143}{2} b^5 c^9 x^{46} + \frac{429}{4} b^6 c^8 x^{44} + \frac{858}{7} b^7 c^7 x^{42} + \frac{429}{4} b^8 c^6 x^{40} + \frac{143}{2} b^9 c^5 x^{38}$$

$$+ \frac{143}{4} b^{10} c^4 x^{36} + 13 b^{11} c^3 x^{34} + \frac{13}{4} b^{12} c^2 x^{32} + \frac{1}{2} b^{13} c x^{30} + \frac{1}{28} b^{14} x^{28}$$

Problem 38: Result more than twice size of optimal antiderivative.

$$\int \frac{x(2cx^2 + b)}{(cx^4 + bx^2)^8} dx$$

Optimal (type 1, 14 leaves, 3 steps):

$$-\frac{1}{14x^{14}(cx^2 + b)^7}$$

Result (type 1, 196 leaves):

$$\frac{c^8 \left(-\frac{b^6}{7c(cx^2 + b)^7} - \frac{66b}{c(cx^2 + b)^2} - \frac{30b^2}{c(cx^2 + b)^3} - \frac{132}{c(cx^2 + b)} - \frac{12b^3}{c(cx^2 + b)^4} - \frac{4b^4}{c(cx^2 + b)^5} - \frac{b^5}{c(cx^2 + b)^6} \right) - \frac{1}{14b^7x^{14}} - \frac{66c^6}{b^{13}x^2} + \frac{33c^5}{b^{12}x^4} - \frac{15c^4}{b^{11}x^6} + \frac{6c^3}{b^{10}x^8} - \frac{2c^2}{b^9x^{10}} + \frac{c}{2b^8x^{12}}}{2b^{13}}$$

Problem 41: Unable to integrate problem.

$$\int \frac{(fx)^m (d + ex^n)}{(a + bx^n + cx^{2n})^2} dx$$

Optimal (type 5, 364 leaves, 5 steps):

$$\frac{(fx)^{1+m} (b^2 d - 2dca - bea + c(-2ae + bd)x^n)}{a(-4ac + b^2)fn(a + bx^n + cx^{2n})} - \frac{1}{a(-4ac + b^2)f(1+m)n(b - \sqrt{-4ac + b^2})} \left(c(fx)^{1+m} \text{hypergeom} \left(\left[1, \frac{1+m}{n} \right], \left[\frac{1+m+n}{n} \right], -\frac{2cx^n}{b - \sqrt{-4ac + b^2}} \right) \left((-2ae + bd)(1+m-n) + \frac{-4acd(1+m-2n) + b^2d(1+m-n) - 2aben}{\sqrt{-4ac + b^2}} \right) \right) - \frac{1}{a(-4ac + b^2)f(1+m)n(b + \sqrt{-4ac + b^2})} \left(c(fx)^{1+m} \text{hypergeom} \left(\left[1, \frac{1+m}{n} \right], \left[\frac{1+m+n}{n} \right], -\frac{2cx^n}{b + \sqrt{-4ac + b^2}} \right) \left((-2ae + bd)(1+m-n) + \frac{4acd(1+m-2n) - b^2d(1+m-n) + 2aben}{\sqrt{-4ac + b^2}} \right) \right)$$

Result (type 8, 31 leaves):

$$\int \frac{(fx)^m (d + ex^n)}{(a + bx^n + cx^{2n})^2} dx$$

Problem 42: Unable to integrate problem.

$$\int \frac{x (d + ex^n)^q}{a + bx^n + cx^{2n}} dx$$

Optimal(type 6, 198 leaves, 5 steps):

$$\frac{cx^2 (d + ex^n)^q \operatorname{AppellF1}\left(\frac{2}{n}, 1, -q, \frac{n+2}{n}, -\frac{2cx^n}{b - \sqrt{-4ac + b^2}}, -\frac{ex^n}{d}\right)}{\left(1 + \frac{ex^n}{d}\right)^q (b^2 - 4ac - b\sqrt{-4ac + b^2})} - \frac{cx^2 (d + ex^n)^q \operatorname{AppellF1}\left(\frac{2}{n}, 1, -q, \frac{n+2}{n}, -\frac{2cx^n}{b + \sqrt{-4ac + b^2}}, -\frac{ex^n}{d}\right)}{\left(1 + \frac{ex^n}{d}\right)^q (b^2 - 4ac + b\sqrt{-4ac + b^2})}$$

Result(type 8, 29 leaves):

$$\int \frac{x (d + ex^n)^q}{a + bx^n + cx^{2n}} dx$$

Problem 43: Unable to integrate problem.

$$\int \frac{(d + ex^n)^q}{a + bx^n + cx^{2n}} dx$$

Optimal(type 6, 186 leaves, 5 steps):

$$\frac{2cx (d + ex^n)^q \operatorname{AppellF1}\left(\frac{1}{n}, 1, -q, 1 + \frac{1}{n}, -\frac{2cx^n}{b - \sqrt{-4ac + b^2}}, -\frac{ex^n}{d}\right)}{\left(1 + \frac{ex^n}{d}\right)^q (b^2 - 4ac - b\sqrt{-4ac + b^2})} - \frac{2cx (d + ex^n)^q \operatorname{AppellF1}\left(\frac{1}{n}, 1, -q, 1 + \frac{1}{n}, -\frac{2cx^n}{b + \sqrt{-4ac + b^2}}, -\frac{ex^n}{d}\right)}{\left(1 + \frac{ex^n}{d}\right)^q (b^2 - 4ac + b\sqrt{-4ac + b^2})}$$

Result(type 8, 28 leaves):

$$\int \frac{(d + ex^n)^q}{a + bx^n + cx^{2n}} dx$$

Problem 44: Unable to integrate problem.

$$\int \frac{x^2 (d + ex^n)^q}{(a + bx^n + cx^{2n})^2} dx$$

Optimal(type 1, 1 leaves, 0 steps):

0

Result(type 8, 31 leaves):

$$\int \frac{x^2 (d + ex^n)^q}{(a + bx^n + cx^{2n})^2} dx$$

Problem 45: Unable to integrate problem.

$$\int \frac{(d + ex^n)^q}{x(a + bx^n + cx^{2n})^2} dx$$

Optimal(type 5, 757 leaves, 0 steps):

$$\begin{aligned} & \frac{(ex+d)^{q+1} (b^2 cd - 2ac^2 d - b^3 e + 3abce + c(2ace - b^2 e + bcd)x)}{a(-4ac + b^2)(ae^2 - bde + cd^2)(cx^2 + bx + a)} - \frac{(ex+d)^{q+1} \operatorname{hypergeom}\left([1, q+1], [2+q], \frac{ex+d}{d}\right)}{a^2 d (q+1)} \\ & - \frac{c(ex+d)^{q+1} \operatorname{hypergeom}\left([1, 1], [1-q], \frac{-2cd + e(b - \sqrt{-4ac + b^2})}{e(b + 2cx - \sqrt{-4ac + b^2})}\right) \left(1 + \frac{b}{\sqrt{-4ac + b^2}}\right)}{a^2 eq (b + 2cx - \sqrt{-4ac + b^2})} \\ & - \frac{1}{a(-4ac + b^2)e(ae^2 - bde + cd^2)q(b + 2cx - \sqrt{-4ac + b^2})} \left(c(ex+d)^{q+1} \operatorname{hypergeom}\left([1, 1], [1-q], \right. \right. \\ & \left. \left. \frac{-2cd + e(b - \sqrt{-4ac + b^2})}{e(b + 2cx - \sqrt{-4ac + b^2})}\right) \left(e(2ace - b^2 e + bcd)q + \frac{-2bc(cd^2 + ae^2(1-2q)) - 4ac^2 deq - b^3 e^2 q + b^2 cde(2+q)}{\sqrt{-4ac + b^2}} \right) \right) \\ & - \frac{c(ex+d)^{q+1} \operatorname{hypergeom}\left([1, 1], [1-q], \frac{-2cd + e(b + \sqrt{-4ac + b^2})}{e(b + 2cx + \sqrt{-4ac + b^2})}\right) \left(1 - \frac{b}{\sqrt{-4ac + b^2}}\right)}{a^2 eq (b + 2cx + \sqrt{-4ac + b^2})} \\ & - \frac{1}{a(-4ac + b^2)e(ae^2 - bde + cd^2)q(b + 2cx + \sqrt{-4ac + b^2})} \left(c(ex+d)^{q+1} \operatorname{hypergeom}\left([1, 1], [1-q], \right. \right. \\ & \left. \left. \frac{-2cd + e(b + \sqrt{-4ac + b^2})}{e(b + 2cx + \sqrt{-4ac + b^2})}\right) \left(e(2ace - b^2 e + bcd)q + \frac{2bc(cd^2 + ae^2(1-2q)) + 4ac^2 deq + b^3 e^2 q - b^2 cde(2+q)}{\sqrt{-4ac + b^2}} \right) \right) \end{aligned}$$

Result(type 8, 31 leaves):

$$\int \frac{(d + ex^n)^q}{x(a + bx^n + cx^{2n})^2} dx$$

Problem 46: Unable to integrate problem.

$$\int \frac{(d + ex^n)^q}{x^2(a + bx^n + cx^{2n})^2} dx$$

Optimal(type 1, 1 leaves, 0 steps):

0

Result(type 8, 31 leaves):

$$\int \frac{(d + ex^n)^q}{x^2 (a + bx^n + cx^{2n})^2} dx$$

Test results for the 7 problems in "1.2.3.5 P(x) (d x)^m (a+b x^n+c x^(2 n))^p.txt"

Problem 1: Result is not expressed in closed-form.

$$\int \frac{mx^8 + lx^7 + kx^6 + jx^5 + hx^4 + gx^3 + x^2f + ex + d}{cx^6 + bx^3 + a} dx$$

Optimal(type 3, 1374 leaves, 37 steps):

$$\begin{aligned} & \frac{kx}{c} + \frac{lx^2}{2c} + \frac{mx^3}{3c} + \frac{(-bm + cj) \ln(cx^6 + bx^3 + a)}{6c^2} + \frac{\ln\left(2^{1/3} c^{1/3} x + \left(b - \sqrt{-4ac + b^2}\right)^{1/3}\right) \left(g - \frac{bk}{c} + \frac{2c^2d + b^2k - c(2ak + bg)}{c\sqrt{-4ac + b^2}}\right) 2^{2/3}}{6c^{1/3} \left(b - \sqrt{-4ac + b^2}\right)^{2/3}} \\ & - \frac{\ln\left(2^{2/3} c^{2/3} x^2 - 2^{1/3} c^{1/3} x \left(b - \sqrt{-4ac + b^2}\right)^{1/3} + \left(b - \sqrt{-4ac + b^2}\right)^{2/3}\right) \left(g - \frac{bk}{c} + \frac{2c^2d + b^2k - c(2ak + bg)}{c\sqrt{-4ac + b^2}}\right) 2^{2/3}}{12c^{1/3} \left(b - \sqrt{-4ac + b^2}\right)^{2/3}} \\ & - \frac{\arctan\left(\frac{\left(1 - \frac{2 \cdot 2^{1/3} c^{1/3} x}{\left(b - \sqrt{-4ac + b^2}\right)^{1/3}}\right) \sqrt{3}}{3}\right) \left(g - \frac{bk}{c} + \frac{2c^2d + b^2k - c(2ak + bg)}{c\sqrt{-4ac + b^2}}\right) 2^{2/3} \sqrt{3}}{6c^{1/3} \left(b - \sqrt{-4ac + b^2}\right)^{2/3}} \\ & - \frac{\ln\left(2^{1/3} c^{1/3} x + \left(b - \sqrt{-4ac + b^2}\right)^{1/3}\right) \left(h - \frac{bl}{c} + \frac{2c^2e + b^2l - c(2al + bh)}{c\sqrt{-4ac + b^2}}\right) 2^{1/3}}{6c^{2/3} \left(b - \sqrt{-4ac + b^2}\right)^{1/3}} \\ & + \frac{\ln\left(2^{2/3} c^{2/3} x^2 - 2^{1/3} c^{1/3} x \left(b - \sqrt{-4ac + b^2}\right)^{1/3} + \left(b - \sqrt{-4ac + b^2}\right)^{2/3}\right) \left(h - \frac{bl}{c} + \frac{2c^2e + b^2l - c(2al + bh)}{c\sqrt{-4ac + b^2}}\right) 2^{1/3}}{12c^{2/3} \left(b - \sqrt{-4ac + b^2}\right)^{1/3}} \\ & + \frac{\arctan\left(\frac{\left(1 - \frac{2 \cdot 2^{1/3} c^{1/3} x}{\left(b - \sqrt{-4ac + b^2}\right)^{1/3}}\right) \sqrt{3}}{3}\right) \left(h - \frac{bl}{c} + \frac{2c^2e + b^2l - c(2al + bh)}{c\sqrt{-4ac + b^2}}\right) 2^{1/3} \sqrt{3}}{6c^{2/3} \left(b - \sqrt{-4ac + b^2}\right)^{1/3}} \end{aligned}$$

$$\begin{aligned}
& \frac{(-2acm + b^2m - bcj + 2c^2f) \operatorname{arctanh}\left(\frac{2cx^3 + b}{\sqrt{-4ac + b^2}}\right)}{3c^2\sqrt{-4ac + b^2}} \\
& + \frac{\ln\left(2^{1/3}c^{1/3}x + (b + \sqrt{-4ac + b^2})^{1/3}\right) \left(g - \frac{bk}{c} + \frac{2ack - b^2k + bcg - 2c^2d}{c\sqrt{-4ac + b^2}}\right) 2^{2/3}}{6c^{1/3}(b + \sqrt{-4ac + b^2})^{2/3}} \\
& - \frac{\ln\left(2^{2/3}c^{2/3}x^2 - 2^{1/3}c^{1/3}x(b + \sqrt{-4ac + b^2})^{1/3} + (b + \sqrt{-4ac + b^2})^{2/3}\right) \left(g - \frac{bk}{c} + \frac{2ack - b^2k + bcg - 2c^2d}{c\sqrt{-4ac + b^2}}\right) 2^{2/3}}{12c^{1/3}(b + \sqrt{-4ac + b^2})^{2/3}} \\
& - \frac{\operatorname{arctan}\left(\frac{\left(1 - \frac{2 \cdot 2^{1/3}c^{1/3}x}{(b + \sqrt{-4ac + b^2})^{1/3}}\right)\sqrt{3}}{3}\right) \left(g - \frac{bk}{c} + \frac{2ack - b^2k + bcg - 2c^2d}{c\sqrt{-4ac + b^2}}\right) 2^{2/3}\sqrt{3}}{6c^{1/3}(b + \sqrt{-4ac + b^2})^{2/3}} \\
& - \frac{\ln\left(2^{1/3}c^{1/3}x + (b + \sqrt{-4ac + b^2})^{1/3}\right) \left(h - \frac{bl}{c} + \frac{2acl - b^2l + bch - 2c^2e}{c\sqrt{-4ac + b^2}}\right) 2^{1/3}}{6c^{2/3}(b + \sqrt{-4ac + b^2})^{1/3}} \\
& + \frac{\ln\left(2^{2/3}c^{2/3}x^2 - 2^{1/3}c^{1/3}x(b + \sqrt{-4ac + b^2})^{1/3} + (b + \sqrt{-4ac + b^2})^{2/3}\right) \left(h - \frac{bl}{c} + \frac{2acl - b^2l + bch - 2c^2e}{c\sqrt{-4ac + b^2}}\right) 2^{1/3}}{12c^{2/3}(b + \sqrt{-4ac + b^2})^{1/3}} \\
& - \frac{\operatorname{arctan}\left(\frac{\left(1 - \frac{2 \cdot 2^{1/3}c^{1/3}x}{(b + \sqrt{-4ac + b^2})^{1/3}}\right)\sqrt{3}}{3}\right) \left(h - \frac{bl}{c} + \frac{2acl - b^2l + bch - 2c^2e}{c\sqrt{-4ac + b^2}}\right) 2^{1/3}\sqrt{3}}{6c^{2/3}(b + \sqrt{-4ac + b^2})^{1/3}}
\end{aligned}$$

Result (type 7, 133 leaves):

$$\begin{aligned}
& \frac{mx^3}{3c} + \frac{lx^2}{2c} + \frac{kx}{c} \\
& + \frac{\sum_{R=\operatorname{RootOf}(Z^6c + Z^3b+a)} \left((-bm + cj) _R^5 + (-bl + ch) _R^4 + (-bk + cg) _R^3 + (-am + cf) _R^2 + (-al + ce) _R - ak + cd \right) \ln(x - _R)}{2 _R^5c + _R^2b} \\
& \frac{3c}{3c}
\end{aligned}$$

Problem 2: Unable to integrate problem.

$$\int \frac{ex+d}{a+bx^n+cx^{2n}} dx$$

Optimal (type 5, 255 leaves, 9 steps):

$$\frac{2cdx \operatorname{hypergeom}\left(\left[1, \frac{1}{n}\right], \left[1 + \frac{1}{n}\right], -\frac{2cx^n}{b - \sqrt{-4ac + b^2}}\right)}{b^2 - 4ac - b\sqrt{-4ac + b^2}} - \frac{cex^2 \operatorname{hypergeom}\left(\left[1, \frac{2}{n}\right], \left[\frac{n+2}{n}\right], -\frac{2cx^n}{b - \sqrt{-4ac + b^2}}\right)}{b^2 - 4ac - b\sqrt{-4ac + b^2}} - \frac{2cdx \operatorname{hypergeom}\left(\left[1, \frac{1}{n}\right], \left[1 + \frac{1}{n}\right], -\frac{2cx^n}{b + \sqrt{-4ac + b^2}}\right)}{b^2 - 4ac + b\sqrt{-4ac + b^2}} - \frac{cex^2 \operatorname{hypergeom}\left(\left[1, \frac{2}{n}\right], \left[\frac{n+2}{n}\right], -\frac{2cx^n}{b + \sqrt{-4ac + b^2}}\right)}{b^2 - 4ac + b\sqrt{-4ac + b^2}}$$

Result (type 8, 24 leaves):

$$\int \frac{ex+d}{a+bx^n+cx^{2n}} dx$$

Problem 3: Unable to integrate problem.

$$\int \frac{ex+d}{(a+bx^n+cx^{2n})^2} dx$$

Optimal (type 5, 718 leaves, 15 steps):

$$\frac{dx(b^2 - 2ac + bcx^n)}{a(-4ac + b^2)n(a + bx^n + cx^{2n})} + \frac{ex^2(b^2 - 2ac + bcx^n)}{a(-4ac + b^2)n(a + bx^n + cx^{2n})} - \frac{2b^2e(-n+2)x^{n+2} \operatorname{hypergeom}\left(\left[1, \frac{n+2}{n}\right], \left[2 + \frac{2}{n}\right], -\frac{2cx^n}{b - \sqrt{-4ac + b^2}}\right)}{a(-4ac + b^2)^{3/2}n(n+2)(b - \sqrt{-4ac + b^2})} + \frac{2b^2e(-n+2)x^{n+2} \operatorname{hypergeom}\left(\left[1, \frac{n+2}{n}\right], \left[2 + \frac{2}{n}\right], -\frac{2cx^n}{b + \sqrt{-4ac + b^2}}\right)}{a(-4ac + b^2)^{3/2}n(n+2)(b + \sqrt{-4ac + b^2})} - \frac{ce(4ac(1-n) - b^2(-n+2))x^2 \operatorname{hypergeom}\left(\left[1, \frac{2}{n}\right], \left[\frac{n+2}{n}\right], -\frac{2cx^n}{b - \sqrt{-4ac + b^2}}\right)}{a(-4ac + b^2)n(b^2 - 4ac - b\sqrt{-4ac + b^2})} - \frac{ce(4ac(1-n) - b^2(-n+2))x^2 \operatorname{hypergeom}\left(\left[1, \frac{2}{n}\right], \left[\frac{n+2}{n}\right], -\frac{2cx^n}{b + \sqrt{-4ac + b^2}}\right)}{a(-4ac + b^2)n(b^2 - 4ac + b\sqrt{-4ac + b^2})}$$

$$\frac{cdx \operatorname{hypergeom}\left(\left[1, \frac{1}{n}\right], \left[1 + \frac{1}{n}\right], -\frac{2cx^n}{b - \sqrt{-4ac + b^2}}\right) \left(4ac(1-2n) - b^2(1-n) - b(1-n)\sqrt{-4ac + b^2}\right)}{a(-4ac + b^2)n(b^2 - 4ac - b\sqrt{-4ac + b^2})}$$

$$\frac{cdx \operatorname{hypergeom}\left(\left[1, \frac{1}{n}\right], \left[1 + \frac{1}{n}\right], -\frac{2cx^n}{b + \sqrt{-4ac + b^2}}\right) \left(4ac(1-2n) - b^2(1-n) + b(1-n)\sqrt{-4ac + b^2}\right)}{a(-4ac + b^2)n(b^2 - 4ac + b\sqrt{-4ac + b^2})}$$

Result(type 8, 201 leaves):

$$\frac{x(ex+d)(-bce^{n \ln(x)} + 2ac - b^2)}{(4ac - b^2)an(a + be^{n \ln(x)} + c(e^{n \ln(x)})^2)} + \int \frac{-bcenxe^{n \ln(x)} + 4acenx - b^2enx - bcdne^{n \ln(x)} + 2bcxe^{n \ln(x)} + 4acd n - 4acex - b^2dn + 2b^2ex + bcd e^{n \ln(x)} - 2dca + b^2d}{(4ac - b^2)an(a + be^{n \ln(x)} + c(e^{n \ln(x)})^2)} dx$$

Problem 4: Unable to integrate problem.

$$\int \frac{-ahx^{-1 + \frac{n}{2}} + cfx^{-1+n} + cgx^{-1+2n} + chx^{-1 + \frac{5n}{2}}}{(a + bx^n + cx^{2n})^{3/2}} dx$$

Optimal(type 3, 71 leaves, 2 steps):

$$\frac{2 \left(c(-2ag + bf) + (-4ac + b^2)hx^{\frac{n}{2}} + c(-bg + 2cf)x^n \right)}{(-4ac + b^2)n\sqrt{a + bx^n + cx^{2n}}}$$

Result(type 8, 59 leaves):

$$\int \frac{-ahx^{-1 + \frac{n}{2}} + cfx^{-1+n} + cgx^{-1+2n} + chx^{-1 + \frac{5n}{2}}}{(a + bx^n + cx^{2n})^{3/2}} dx$$

Problem 5: Unable to integrate problem.

$$\int \frac{(dx)^{-1 + \frac{n}{2}} \left(-ah + cfx^{\frac{n}{2}} + cgx^{\frac{3n}{2}} + chx^{2n} \right)}{(a + bx^n + cx^{2n})^{3/2}} dx$$

Optimal(type 3, 87 leaves, 2 steps):

$$\frac{2x^{1 - \frac{n}{2}} (dx)^{-1 + \frac{n}{2}} \left(c(-2ag + bf) + (-4ac + b^2)hx^{\frac{n}{2}} + c(-bg + 2cf)x^n \right)}{(-4ac + b^2)n\sqrt{a + bx^n + cx^{2n}}}$$

Result(type 8, 57 leaves):

$$\int \frac{(dx)^{-1+\frac{n}{2}} \left(-ah + cfx^{\frac{n}{2}} + cgx^{\frac{3n}{2}} + chx^{2n} \right)}{(a + bx^n + cx^{2n})^{3/2}} dx$$

Problem 6: Unable to integrate problem.

$$\int (gx)^m (a + bx^n + cx^{2n})^p (a(1+m) + b(np+m+n+1)x^n + c(1+m+2n(1+p))x^{2n}) dx$$

Optimal(type 3, 29 leaves, 1 step):

$$\frac{(gx)^{1+m} (a + bx^n + cx^{2n})^{1+p}}{g}$$

Result(type 8, 58 leaves):

$$\int (gx)^m (a + bx^n + cx^{2n})^p (a(1+m) + b(np+m+n+1)x^n + c(1+m+2n(1+p))x^{2n}) dx$$

Problem 7: Unable to integrate problem.

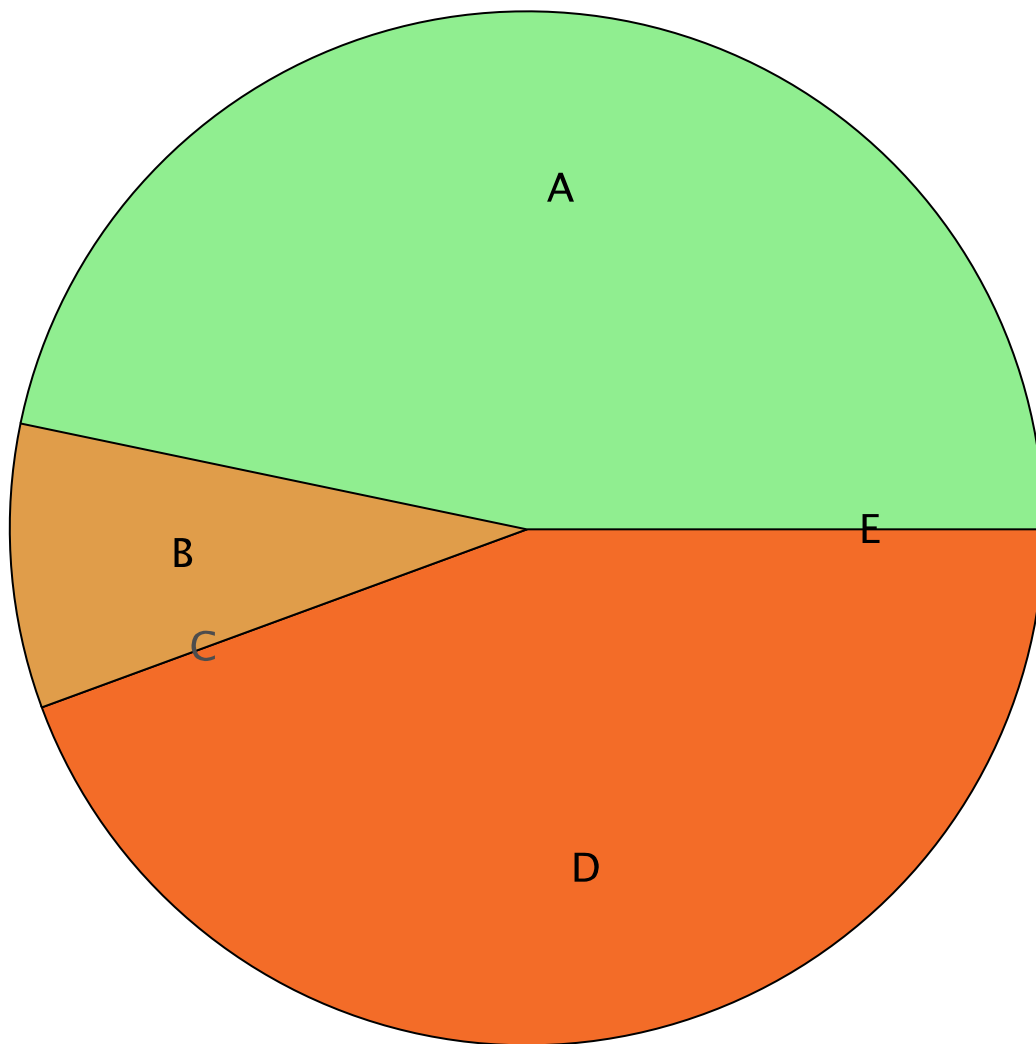
$$\int \frac{A + Bx^n + Cx^{2n} + Dx^{3n}}{(a + bx^n + cx^{2n})^2} dx$$

Optimal(type 5, 485 leaves, 4 steps):

$$\begin{aligned} & \frac{x(Ac(-2ac+b^2) - a(bBc - 2acC + abD) + (bc(Ac + aC) - ab^2D - 2ac(Bc - aD))x^n)}{ac(-4ac+b^2)n(a+bx^n+cx^{2n})} \\ & + \frac{1}{ac(-4ac+b^2)n(b-\sqrt{-4ac+b^2})} \left(x \operatorname{hypergeom} \left(\left[1, \frac{1}{n} \right], \left[1 + \frac{1}{n} \right], -\frac{2cx^n}{b-\sqrt{-4ac+b^2}} \right) \left(ab^2D - bc(Ac + aC)(1-n) + 2ac(Bc(1-n) - aD(1+n)) + \frac{Ac^2(4ac(1-2n) - b^2(1-n)) - a(4ac^2C + b^3D - b^2cC(1-n) - 2bc(Bcn + aD(n+2)))}{\sqrt{-4ac+b^2}} \right) \right) \\ & + \frac{1}{ac(-4ac+b^2)n(b+\sqrt{-4ac+b^2})} \left(x \operatorname{hypergeom} \left(\left[1, \frac{1}{n} \right], \left[1 + \frac{1}{n} \right], -\frac{2cx^n}{b+\sqrt{-4ac+b^2}} \right) \left(ab^2D - bc(Ac + aC)(1-n) + 2ac(Bc(1-n) - aD(1+n)) + \frac{-Ac^2(4ac(1-2n) - b^2(1-n)) + a(4ac^2C + b^3D - b^2cC(1-n) - 2bc(Bcn + aD(n+2)))}{\sqrt{-4ac+b^2}} \right) \right) \end{aligned}$$

Result(type 8, 40 leaves):

$$\int \frac{A + Bx^n + Cx^{2n} + Dx^{3n}}{(a + bx^n + cx^{2n})^2} dx$$



A - 121 optimal antiderivatives
B - 23 more than twice size of optimal antiderivatives
C - 0 unnecessarily complex antiderivatives
D - 115 unable to integrate problems
E - 0 integration timeouts